ECE 640 Project # 2: On Image Reconstruction from Projections
(Issued Tuesday 11/03, Due Thursday 12/9)

Purpose: The purpose of this project is to study image reconstruction from projection using the
filtered-backprojection approach.

In this experiment we will obtain an estimate of an image by filtered-backprojection of parallel data. We will use the Shepp and Logan head-phantom image described in A. C. Kak, “Image Reconstruction from Projection,” Chap. 4 in M. P. Ekstrom, Digital Image Processing Techniques, Academic Press, New York, 1984, pp. 111-170. This model is formed of 10 ellipses,

\[ g_i(x, y) = \begin{cases} \rho_i & \text{for } x^2/A_i^2 + y^2/B_i^2 \leq 1 \quad \text{(inside the ellipse)} \\ 0 & \text{otherwise} \quad \text{(outside the ellipse).} \end{cases} \]

The values of \(A_i, B_i, \rho_i, \ i \in [1, 10]\) are shown in Table I of the above reference. It is straightforward to show that the projections of each of these ellipses is as follows

\[ P_\theta(t) = \begin{cases} \frac{2\rho_i A_i B_i \sqrt{a_i^2(\theta) - t^2}}{a_i^2(\theta)} & \text{for } |t| \leq a_i(\theta) \\ 0 & \text{for } |t| > a_i(\theta) \end{cases} \]

where \(a_i(\theta) = A_i^2 \cos^2 \theta + B_i^2 \sin^2 \theta\) is the projection half-width.

You are required to do the following:

1. Obtain a parallel-beam reconstruction along the lines of Fig. 7 of the reference.
2. Study the aliasing effects on the reconstruction and obtain a plot quantifying this effect along the lines of Fig. 21 of the reference.

You may use the same number of rays and/or projections as in the reference.

Note: Submit an intelligent and concise report that includes an overall description of the project, the theoretical ideas in each part, and comments on the results (accuracy and/or lack off with full reasoning).