1. \( P_n(x) = \frac{1}{n} \sum_{i=1}^{n} \Phi\left( \frac{x-x_i}{\sqrt{n}} \right) \quad n = 20 \quad h_n = \frac{1}{\sqrt{20}} \\
\Phi\left( \frac{x-x_i}{\sqrt{20}} \right) = e^{-\frac{1}{2} \left( x-x_i \right)^2}
\)
\[ P_n(x) = \frac{1}{20} \sum_{i=1}^{20} e^{-\sqrt{20}(x-x_i)} \]
\[ P_n(x) = \frac{1}{\sqrt{20}} \left( \frac{20}{\sqrt{20}} e^{-\sqrt{20}(x-x_i)} \right) = \frac{1}{\sqrt{20}} \left( 2e^{-\sqrt{20}x} + 14e^{-\sqrt{20}(x+2)} + 4e^{-\sqrt{20}(x-1)} \right) \]

(b) \( P(\mid x \mid \leq 1) = \int_{-1}^{1} P(x) \, dx = \frac{1}{\sqrt{20}} \left[ e^{-\sqrt{20}x} + 14e^{-\sqrt{20}(x+2)} + 4e^{-\sqrt{20}(x-1)} \right] \]
\[ = \frac{1}{\sqrt{20}} \left[ \frac{2}{\sqrt{20}} e^{-\sqrt{20}} + 2e^{-\sqrt{20}} - \frac{14}{\sqrt{20}} e^{-\sqrt{20}} + \frac{14}{\sqrt{20}} e^{-\sqrt{20}} - \frac{4}{\sqrt{20}} e^{-\sqrt{20}} \right] \]
\[ = \frac{1}{20} \left[ -2e^{-\sqrt{20}} - 12e^{-\sqrt{20}} + 14e^{-\sqrt{20}} - 4 + 4 \frac{2}{\sqrt{20}} \right] \approx 0.90 \]
\( x_a = x_p + r \frac{w}{\|w\|^2} \)

\[ g(x) = w^T x + w_d = w^T (x_p + r \frac{w}{\|w\|^2}) + w_d \]

\[ g(x) = w^T x_p + w_d + r \frac{ww^T}{\|w\|^2} \]

\[ g(x) = r \frac{\|w\|^2}{\|w\|^2} = \frac{r}{\|w\|} \]

\[ \text{distance} = R = \frac{g(x)}{\|w\|} = \frac{w^T x_a + w_d}{\|w\|} \]

(b) \( x_a = x_p + r \frac{w}{\|w\|^2} \)

\( x_p = x_a - r \frac{w}{\|w\|^2} \), since \( R = \frac{g(x)}{\|w\|} \)

\[ x_p = x_a - \frac{g(x_a)}{\|w\|^2} \frac{w}{w} \]
\( J_r(a) = \frac{1}{2} \sum_{y \in Y} \frac{(a^T y - b)^2}{\|y\|^2} \)

\[
\nabla J_r(a) = \frac{d}{da} J_r(a) = \sum_{y \in Y} \frac{a^T y - b}{\|y\|^2} y
\]

So, update equation for batch method, with learning rate \( \eta \)

\[
a(k+1) = a(k) + \eta (k) \sum_{y \in Y} \frac{b - a^T y}{\|y\|^2} y
\]
\[ a = (y^T y)^{-1} y^T b \]

\[ y_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad y_2 = \begin{bmatrix} 0 \\ -3 \end{bmatrix}, \quad y_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad y_4 = \begin{bmatrix} -2 \\ -3 \end{bmatrix} \]

\[ y^T = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & -3 & -3 \end{bmatrix} \]

\[ y^T y = \begin{bmatrix} 3/2 & 18/11 \\ 18/11 & 11/4 \end{bmatrix}, \quad (y^T y)^{-1} = \begin{bmatrix} 23/18 & -13/18 & -1/3 \\ -13/18 & 1/9 & 2/9 \\ -1/3 & 2/9 & 1/4 \end{bmatrix} \]

\[ (y^T y)^{-1} y^T b = \begin{bmatrix} 11/3 \\ -4/3 \\ -2/3 \end{bmatrix}, \quad b > \begin{bmatrix} 1 \\ 1 \end{bmatrix} \]

Check:
\[ a^T y_1 = 1 > 0 \Rightarrow w_1 \]
\[ a^T y_2 = 0 \Rightarrow w_2 \]
\[ a^T y_3 = -1 < 0 \Rightarrow w_2 \]
\[ a^T y_4 = -1 < 0 \Rightarrow w_2 \]

So, \[ a = \begin{bmatrix} 11/3 \\ -4/3 \\ -2/3 \end{bmatrix} \] classifier is \( w_1 \) if \( a^T y > 0 \) \( w_2 \) if \( a^T y < 0 \)