1. In the two-category case, under the Bayes decision rule the conditional error is given by $P(error/x) = \min\{P(\omega_1/x), P(\omega_2/x)\}$. Show that

$$P(error/x) < 2P(\omega_1/x)P(\omega_2/x)$$

2. Suppose two equally probable one-dimensional densities are of the form $p(x/\omega_i) \propto e^{-|x-a_i|^2/b_i}$ for $i=1,2$ and $b_i > 0$.
   a) Write an analytic expression for each density.
   b) Calculate the likelihood ratio.

3. Consider the following decision rule for a two category one-dimensional problem:
   decide $\omega_1$ if $x > \theta$; otherwise decide $\omega_2$.
   a) Show that the probability of error is given by

   $$P(error) = P(\omega_1)\int_{-\infty}^{\theta} p(x/\omega_1)dx + P(\omega_2)\int_{\theta}^{\infty} p(x/\omega_2)dx$$

   b) Show that a necessary condition to minimize $P(error)$ is that

   $$p(\theta/\omega_1)P(\omega_1) = p(\theta/\omega_2)P(\omega_2)$$

   **Note:** Libnitz Rule: $\frac{d}{dz} \int_{U(z)}^{U(z)} f(x) dx = f(U_2(z)) \frac{d}{dz} U_2(z) - f(U_1(z)) \frac{d}{dz} U_1(z)$

4. Consider the multivariate normal density with mean $\mu$, and $\Sigma = diag(\sigma_1^2, \sigma_2^2, ..., \sigma_d^2)$.
   a) Show that the evidence is given by:

   $$p(x) = \frac{1}{\prod_{i=1}^{d} \sqrt{2\pi\sigma_i}} \exp\left[ -\frac{1}{2} \sum_{i=1}^{d} \left( \frac{x_i - \mu_i}{\sigma_i} \right)^2 \right]$$

   b) Write an expression for the Mahalanobis distance from $x$ to $\mu$ given by

   $$(x - \mu)^T \Sigma^{-1} (x - \mu).$$

5. Derive an expression for the discriminant functions for the multivariate normal density with equal covariance matrices (i.e. $\Sigma_i = \Sigma$).