ECE 643: Introduction to Biocomputing

Homework 1
(Assigned Tuesday 1/20/09 Due Tuesday 1/27/09)

1. Let \( q, v \) and \( r \) be quaternions.
   a. Prove that quaternion multiplication preserves dot product, i.e. \((qv)\cdot(qr) = (q.q)\cdot(v.r)\). What happens in case \( q \) is a unit quaternion?
   b. Prove that the composite product leads to purely imaginary quaternion, hence it can be used to represent rotation, i.e. Rotating the vector (point) \( r \) by a unit quaternion \( q \) can be defined as \( r' = qrq^* \), where \( r' \) is a purely imaginary quaternion representing the vector \( r \) after rotation by \( q \).
   c. Prove that the unit quaternion that maximizes \( q^TNq \) is the eigenvector corresponding to the most positive eigenvalue of the matrix \( N \).

2. Consider the matrix.
   \[
   A = \begin{bmatrix}
   10 & -18 \\
   6 & -11
   \end{bmatrix}
   \]
   Calculate the eigenvalues and the corresponding eigenvectors.

3. Given the following ensemble (collection/measurement) of 2D points.
   \[
   X = \begin{bmatrix}
   1 & 1 & 1 & 2 & 3 & 3 & 2 & 3 & 3 & 5 & 5 & 5 \\
   1 & 2 & 4 & 1 & 1 & 3 & 2 & 2 & 4 & 1 & 4 & 5
   \end{bmatrix}
   \]
   a. Compute the centroid
   b. Calculate the covariance matrix
   c. Compute the eigenvalues and eigenvectors of the covariance matrix found in (b)

4. Affine transformation. Consider the following matrices. As discussed in class, any three-dimensional affine transformation can be represented with a 4x4 matrix.
   a. Match each of the matrices above to exactly one of the following transformations (not all blanks will be filled):
      __ Differential (Non-Uniform) Scaling
      __ Reflection
      __ Rotation about the z-axis with non-uniform scaling
      __ Rotation about the y-axis with non-uniform scaling
      __ Translation
      __ Rotation about the x-axis
      __ Rotation about the y-axis
      __ Rotation about the z-axis
      __ Shearing along \( z \) with respect to the x-y plane (\( z=0 \) plane unchanged by shear)
      __ Shearing along \( x \) with respect to the y-z plane (\( x=0 \) plane unchanged by shear)
      __ Rotation about the x-axis and translation
      __ uniform scaling
      __ Reflection with uniform scaling
b. Extra Credit: Consider a line that passes through a point \( p = (p_x, p_y, p_z) \) in the direction \( v = (\cos \alpha, 0, \sin \alpha) \). Write out the product of matrices that would perform a rotation by \( \theta \) about this line. You should not multiply these matrices out, but you do need to write out all of the elements in these matrices.