Sources of Uncertainty

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Outline

- Definitions
  - Spatial Resolution
  - Contrast Resolution
- Imaging Artifacts
  - Partial Volume Effect
  - Bias Field (Intensity Nonuniformity Field)
  - Beam Hardening
  - Ring Effect
  - Motion
- Accuracy
Resolution has two components, spatial resolution and contrast resolution.

- **Spatial Resolution**: The ability of an imaging unit to display, as separate images, two objects that are very close to each other.

- **Contrast Resolution**: The ability of an imaging unit to display, as distinct images, areas that differ in density by a small amount.
Introduction

- Attempts to improve *spatial resolution* may cause increased noise levels that decrease contrast resolution.

- Attempts to improve *contrast resolution* by increasing scan times (larger dose) may decrease spatial resolution by increasing the effects of patient motion.
Spatial Resolution

- Spatial resolution measurements are determined by using test objects of high contrast.

- High contrast test objects make it possible to ignore the effect of noise on spatial resolution measurements.

- High contrast spatial resolution is determined by
  - The scanner design,
  - The computer reconstruction, and
  - The display (Field of view)
Spatial Resolution

- Scanner design includes x-ray tube focal spot size, detector size, and magnification.

- Detector size influences spatial resolution of a CT scanner. Detector size places a limit on maximum spatial resolution.

- To increase resolution, it is possible to put an aperture in front of a detector and effectively reduce detector size.
Contrast Resolution

- For the image to be visible, the object must produce enough change in the number of transmitted photons to overcome fluctuations in transmitted photons caused by noise.

- Low contrast visibility is determined by noise. The more homogenous background, the better the visibility of low contrast images.
Factors Affecting Spatial Resolution

- **Detector pitch:** The detector pitch is the center-to-center spacing of the detectors along the array. For third-generation scanners, the detector pitch determines the ray spacing.

- **Number of rays:** The number of rays used to produce a CT image over the same Field of view (FOV) has a strong influence on spatial resolution.

- **Slice thickness:** The slice thickness is equivalent to the detector aperture in the cranial-caudal axis. Large slice thickness reduce spatial resolution in the cranial-caudal axis, but they also reduce sharpness of the edges of structures.
Factors Affecting Spatial Resolution

- **Helical pitch:** The pitch used in helical CT scanning affects spatial resolution with greater pitches reducing resolution.

- **Pixel matrix:** The number of pixels used to reconstruct the CT image has a direct influence on spatial resolution.

- **Patient motion:** Image will experience blurring proportional to the distance of the motion during the scan.

- **Field of view:** The FOV influences the physical dimensions of each pixel.
Factors Affecting Contrast Resolution

- **mAs:** The mAs directly influence the number of x-ray photons used to produce the CT image, thereby affecting the SNR and the contrast resolution. Doubling of the mAs of the study increases the SNR by $\sqrt{2}$ and the contrast resolution consequently improves.

- **Pixel size (FOV):** If patient size and all other scan parameters are fixed, as FOV increases, pixel dimensions increase, and the number of x-rays passing through each pixel increases.

- **Slice thickness:** The slice thickness has a strong influence on the number of photons used to produce the image. Thicker slice use more photons and have better SNR.
Factors Affecting Contrast Resolution

- **Reconstruction filter:** Bone filters produce lower contrast resolution, and soft tissue filters improve contrast resolution.

- **Patient size:** For the same x-ray technique, larger patients attenuate more x-rays, resulting in detection of fewer x-rays. This reduces the SNR and therefore the contrast resolution.

- **Gantry rotation speed:** Most CT systems have an upper limit on mA, and for a fixed mA, faster gantry rotations result in reduces mAs used to produce each CT image, reducing contrast resolution.
Image Quality

- Image quality depends on the particular imaging modality used. For each modality, the range of image quality may be considerable, depending on the characteristics and set up of the particular medical imaging system. Following six important factors effect image quality:
  - **Contrast**: High contrast allows easier identification of individual objects.
  - **Resolution**: The ability of an imaging system to depict details.
  - **Noise**: An image may be corrupted by random fluctuations in image intensity.
  - **Artifacts**: Obscure important features, or falsely interpreted as abnormal findings.
  - **Distortion**: Changes of shape, size, position, and other geometric characteristics.
  - **Accuracy**: The quality of medical images should be judged.
Image Quality: Contrast

- **Modulation:** Use of a periodic signal and its modulation is an effective way to quantify contrast. The modulation $m_f$ of a periodic signal $f(x, y)$ with minimum and maximum values $f_{\text{max}}$ and $f_{\text{min}}$ is defined by

$$m_f = \frac{f_{\text{max}} - f_{\text{min}}}{f_{\text{max}} + f_{\text{min}}}$$

In general, $m_f$ refers to the contrast of the periodic signal $f(x, y)$ relative to its average value.

$m_f = 1$ only when $f_{\text{min}} = 0$.

If $f(x, y)$ and $g(x, y)$ are two periodic signals with the same average value, we say that $f(x, y)$ has more contrast than $g(x, y)$ if $m_f > m_g$. 

Image Quality: Contrast

- **Local Contrast**: The identification of some specific object or feature within an image is only possible if its value differs from that of surrounding areas.

- Suppose that a Region of Interest (ROI) or target has a nominal image intensity of $f_t$. The ROI is surrounded by other tissues, called the background with nominal intensity of $f_b$.

- The difference between the target and its background is captures by the local contrast, defined as

$$C = \frac{f_t - f_b}{f_b}$$
Consider an image showing an organ with intensity $I_0$ and a tumor with intensity $I_t > I_0$. What is the local contrast of the tumor? If we add a constant intensity $I_c > 0$ to the image, what is the local contrast? Is the contrast improved?
Image Quality: Contrast

- **Local Contrast-Answer:**

  By definition, the local contrast of the tumor is

  \[ C = \frac{I_t - I_0}{I_0} \]

  If we add a constant intensity \( I_c \) to the image, the intensities of the background and the target become \( f_b = I_0 + I_c \), and \( f_t = I_t + I_c \).

  The local contrast of the proposed image is

  \[ C' = \frac{(I_t + I_c) - (I_0 + I_c)}{I_0 + I_c} = \frac{I_t - I_0}{I_0 + I_c} = C \frac{I_0}{I_0 + I_c} < C \]

  So the local contrast is worse if we add a constant intensity \( I_c \) to the image.
Image Quality: Random Variables

- **Random Variables:** The numeric quantity associated with a random event or experiment is called a random variable. Different repetitions of the experiment may produce different observed values, i.e., the experiment has a random outcome.

- A random variable is mathematically described by $P_N(\eta)$, its **probability distribution function** (PDF) given by

\[
P_N(\eta) = \Pr[N \leq \eta]
\]

where $\Pr[.]$ denotes probability.
Image Quality: Random Variables

- A random variable is mathematically described by $P_N(\eta)$, its probability distribution function (PDF) given by

$$P_N(\eta) = \Pr[N \leq \eta]$$

- The PDF gives the probability that random variable $N$ will take on a value less than or equal to $\eta$.

- Notice that

$$0 \leq P_N(\eta) \leq 1$$
$$P_N(-\infty) = 0$$
$$P_N(\infty) = 1$$
$$P_N(\eta_1) \leq P_N(\eta_2) \quad \text{for} \quad \eta_1 \leq \eta_2$$
**Image Quality: Random Variables**

- **Continuous Random Variables:** If \( P_N(\eta) \) is a continuous function of \( \eta \) then \( N \) is a continuous random variable. This random variable is uniquely specified by its probability density function (pdf),

\[
p_N(\eta) = \frac{dP_N(\eta)}{d\eta}
\]

Any pdf satisfies the following three properties:

\[
P_N(\eta) \geq 0
\]

\[
\int_{-\infty}^{\infty} p_N(\eta)d\eta = 1
\]

\[
P_N(\eta) = \int_{-\infty}^{\eta} p_N(u)du
\]
Continuous Random Variables: In practice, the pdf of a random variable may not be known. Instead, a random variable is often characterized by its expected value or mean.

\[ \mu_N = E[N] = \int_{-\infty}^{\infty} \eta p_N(\eta) d\eta \]

Its variance

\[ \sigma_N^2 = Var[N] = E[(N - \mu_N)^2] = \int_{-\infty}^{\infty} (\eta - \mu_N)^2 p_N(\eta) d\eta \]

where \( \sigma_N \) is called the standard deviation of \( N \).
Image Quality: Random Variables

Uniform Random Variable: A random variable $N$ is said to be uniform over interval $[a, b]$ if its pdf of the form.

$$p_N(\eta) = \begin{cases} 1/(b-a) & \text{for } a \leq \eta < b \\ 0 & \text{o.w.} \end{cases}$$

In this case the distribution function is given by

$$P_N(\eta) = \begin{cases} 0 & \text{for } \eta < a \\ (\eta-a)/(b-a) & \text{for } a \leq \eta \leq b \\ 1 & \text{for } \eta > b \end{cases}$$

$$\mu_N = \frac{a+b}{2}$$

$$\sigma_N^2 = \frac{(b-a)^2}{12}$$
Image Quality: Random Variables

**Gaussian Random Variable:** In the pdf of a random variable \( N \) is given by

\[
    p_N(\eta) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(\eta-\mu)^2}{2\sigma^2}}
\]

In this case the distribution function is given by

\[
    P_N(\eta) = \frac{1}{2} + \text{erf} \left( \frac{\eta - \mu}{\sigma} \right)
\]

where \( \text{erf}(x) \) denotes the error function, given by the integral

\[
    \text{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-u^2/2} du
\]

\[\mu_N = \mu\]
\[\sigma_N^2 = \sigma^2\]
Discrete Random Variables: When the random variable $N$ takes only values $\eta_1, \eta_2, \ldots, \eta_k$, it is said to be a discrete random variable. This random variable is uniquely specified by the probability mass function (PMF) $\Pr[N = \eta_i]$, for $i = 1, 2, \ldots, k$.

The PMF satisfies the following three properties:

- $0 \leq \Pr[N = \eta_i] \leq 1$, for $i = 1, 2, \ldots, k$

- $\sum_{i=1}^{k} \Pr[N = \eta_i] = 1$

- $P_N(\eta) = \Pr[N \leq \eta] = \sum_{\eta_i \leq \eta} \Pr[N = \eta_i]$
Image Quality: Random Variables

Discrete Random Variables: The mean value and variance are given by

\[ \mu_N = E[N] = \sum_{i=1}^{k} \eta_i \Pr[N = \eta_i] \]

and

\[ \sigma_N^2 = Var[N] = E[(N - \mu_N)^2] = \sum_{i=1}^{k} (\eta_i - \mu_N)^2 \Pr[N = \eta_i] \]
Image Quality: Random Variables

**Independent Random Variables:** It is usual in imaging experiments to consider more than one random variable at a time.

Consider the collection of random variables $N_1, N_2, \ldots, N_m$, having the pdf’s $p_1(\eta), p_2(\eta), \ldots, p_m(\eta)$, respectively. The sum of these random variables $S$ is another random pdf, $p_s(\eta)$. It is always the case that the mean of $S$ is precisely the sum of the means of $N_1, N_2, \ldots, N_m$. That is,

$$\mu_s = \mu_1 + \mu_2 + \ldots + \mu_m$$

$$\sigma_s^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_m^2$$

Also it is possible to determine the pdf of $S$ by convolution.

$$p_s(\eta) = p_1(\eta) \ast p_2(\eta) \ast \ldots \ast p_m(\eta)$$
**Image Quality: Signal-to-Noise Ratio**

- **SNR:** Assume that the output of a medical imaging system is a random variable $G$, composed of two components, $f$ and $N$. Component $f$, which is usually referred to as *signal*, is the “true” value of $G$, whereas $N$ is a random fluctuation or error component due to noise.

- The SNR describes the relative “strength” of signal $f$ with respect to that of noise $N$.

- *Higher SNR* values indicate that $g$ is a more accurate representation of $f$. 
Image Quality: Signal-to-Noise Ratio

Amplitude SNR: Most frequently, the SNR is expressed as the ratio of signal amplitude to noise amplitude:

\[
SNR_a = \frac{Amplitude(f)}{Amplitude(N)}
\]

Example: In projection radiography, the number of photons G counted per unit area by an x-ray image intensifier follows a Poisson distribution. In this case, we may consider signal f to be average photon count per area (i.e., the mean of G) and noise N to be the random variation of this count around the mean, whose amplitude is quantified by standard deviation of G.

What is the amplitude SNR of such a system?
Answer: $G$ is a random variable that follows the Poisson Distribution.

$$\Pr[G = k] = \frac{a^k}{k!} e^{-a} \quad \text{for} \quad k = 0, 1, \ldots,$$

Where $a > 0$ is a real-valued parameter. The mean and variance of $G$, Poisson random variable are:

$$\mu_G = a \quad \quad \sigma_N^2 = a$$

So amplitude SNR is given by

$$SNR_a = \frac{\mu_G}{\sigma_G} = \frac{\mu}{\sqrt{\mu}} = \sqrt{\mu}$$
Image Quality: Signal-to-Noise Ratio

- **Power SNR:** Another way to express the SNR is as the ratio of signal power to noise power:

\[
SNR_p = \frac{\text{power}(f)}{\text{power}(N)}
\]

- **Differential SNR:** Consider an object (target) of interest placed on a background. Let \( f_t \) and \( f_b \) be the average image intensities within the target and background, respectively.

Let \( A \) be some region of the image area. Then

\[
SNR_{\text{diff}} = \frac{A(f_t - f_b)}{\sigma_b(A)}
\]
**Image Quality: Signal-to-Noise Ratio**

**Decibels:** The SNR is sometimes given in *decibels (dB).*

\[
\text{SNR \ (in \ dB)} = 20 \times \log_{10}(\text{SNR}) \quad \text{(ratio of amplitudes)}
\]

\[
\text{SNR \ (in \ dB)} = 10 \times \log_{10}(\text{SNR}) \quad \text{(ratio of power)}
\]
Image Quality: Signal-to-Noise Ratio

**SNR:** We can express the SNR in more detail by adding several addition concepts.

Since we indicate above that SNR is proportional to the number of photons per unit area of detector, if we increase the unit area, we will increase the number of photons. Therefore, the detailed expression for SNR becomes

$$SNR = C \sqrt{\Phi ARt \eta}$$

where \( \Phi \) is the number of photons per Roentgen per \( cm^2 \), \( A \) is the unit area, \( R \) is the body’s radiation exposure in roentgens, \( t \) is the fraction of photons transmitted through the body, \( \eta \) is the detector efficiency.
**Image Quality: Signal-to-Noise Ratio**

**Example:** Consider the following parameters, which are from a typical chest x-ray:

\[ \Phi = 637 \times 10^6 \text{ photons per } R^{-1} \text{ cm}^{-2} \]

\[ R = 50 \text{ mR} \]

\[ t = 0.05 \]

\[ \eta = 0.25 \text{(efficiency)} \]

\[ A = 1 \text{ mm}^2 \]

What is the SNR of a lesion having 10% contrast, i.e., C=0.1?

\[ SNR = 16 \text{ dB} \]
Image Quality: Compton Scattering

We noted that Compton scattering degrades image quality. The reason for this is that Compton photons are deflected from their ideal straight-line path, and some are detected in locations away from the correct, straight-line location. This produces two unwanted results: a decrease in image contrast and a decrease in SNR.

Effect on image contrast: Recall that local contrast was defined as

\[ C = \frac{I_t - I_0}{I_0} \]

Compton scatter adds a constant intensity \( I_s \) to both target and background intensity, yielding a new contrast of

\[ C' = \frac{(I_t + I_s) - (I_b + I_s)}{I_b + I_s} = C \frac{I_b}{I_b + I_s} = \frac{C}{1 + I_s/I_b} \]
Image Quality: Compton Scattering

Effect on image contrast: Therefore, the effect of scatter is to reduce contrast by factor \( \frac{1}{1 + \frac{I_s}{I_b}} \). The ratio \( \frac{I_s}{I_b} \) is called scatter-to-primary ratio.
Image Quality: Compton Scattering

Effect on Signal-to-Noise Ratio with Scatter: The derivation of SNR in the presence of Compton scattering follows the Compton-free derivation very closely:

\[
SNR' = \frac{I_t - I_b}{\sigma_b} = C \frac{I_b}{\sigma_b} = C \frac{N_b}{\sqrt{N_b + N_s}} = C \frac{\sqrt{N_b}}{\sqrt{1 + N_b / N_s}}
\]

Here, the symbol \( N_s \) stands for the number of Compton scattered photons per burst per area \( A \) on the detector, and the symbol \( C \) is the underlying contrast (not the Compton scatter-reduced contrast).
**Image Quality: Compton Scattering**

**Example:** Suppose 20 percent of the incident x-ray photons have been scattered in a certain material before they arrive at detectors.

What is the scatter-to-primary ratio? By what factor is the SNR degraded?

**Answer:** The x-ray photons that contribute to the background intensity are those that hit the detectors without Compton scattering. The number of these photons is $0.8N$, where $N$ is the number of incident x-ray photons. The number of scattered photons is $0.2N$. Because the intensity of the image is proportional to the number of photons detected, we have

$$I_b \propto 0.8N \quad I_s \propto 0.2N$$
Image Quality: Compton Scattering

Answer: The scatter-to-primary ratio is

\[
\frac{I_s}{I_b} = \frac{0.2N}{0.8N} = \frac{1}{4}
\]

We can see that the loss of SNR is \(1 - \frac{1}{1 + \frac{I_s}{I_b}}\) = 0.11

So the Compton scattering introduces 11 percent loss of SNR.
Image Quality: Contrast

- **Local Contrast-Answer:**

  By definition, the local contrast of the tumor is

  \[ C = \frac{I_t - I_0}{I_0} \]

  If we add a constant intensity \( I_c \) to the image, the intensities of the background and the target become \( f_b = I_0 + I_c \), and \( f_t = I_t + I_c \).

  The local contrast of the proposed image is

  \[ C' = \frac{(I_t + I_c) - (I_0 + I_c)}{I_0 + I_c} = \frac{I_t - I_0}{I_0 + I_c} = C \frac{I_0}{I_0 + I_c} < C \]

  So the local contrast is worse if we add a constant intensity \( I_c \) to the image.
CT has the best contrast resolution of any clinical x-ray modality.

Contrast resolution is fundamentally tied to the SNR.

The SNR is also very much related to the number of x-ray quanta used per pixel in the image.

Reducing pixel size results in increasing spatial resolution and reducing the number of x-rays per pixel is reduced.

In CT there is a well-established relationship among SNR, pixel dimension ($\Delta$), slice thickness ($T$), and radiation dose ($D$):

$$D \propto \frac{SNR^2}{\Delta^3 T}$$
Imaging Artifacts

- Partial Volume Effect
- Bias Field (Intensity Nonuniformity Field)
- Beam Hardening
- Ring Effect
- Motion
Partial Volume Effect

- Partial volume effects (PVEs) occur due to the finite limits of the image resolution.
Partial Volume Effect

- Partial volume effects (PVEs) occur due to the finite limits of the image resolution.
- PVE occurs when some pixels in the image contain a mixture of different types.
- When this occur, for example with bone and soft tissue, the attenuation coefficient $\mu$ is a weighted average of the two different $\mu$ values.
- PVEs can lead to misdiagnosis when the presence of adjacent anatomic structures is not suspected.
Partial Volume Effect

- Partial volume averaging becomes problematic when assessing small lesions or small structures.
- Small or thin structures entirely contained within a voxel may disappear following volume averaging.
- Also, PVE causes edge blurring.
- The solutions:
  - Increasing the spatial resolution
    - This often come at the expense of SNR, time or both.
  - Using a thin collimation.
Partial Volume Effect

- For more information read following references for partial volume correction.


Partial Volume Effect

Salvado proposed a method to restore the ideal boundary by splitting a voxel into subvoxels and reapportioning the signal into the subvoxels. Here is an example in the figure.

![Example Images](image)

**Figure 5:** Original $100 \times 100$ voxels Lena picture (a) is interpolated with the proposed method two times (b) and four times (c) as well as four times with bicubic interpolation (d). Lower row of panels show a zoom on the left eye of the corresponding top row images. The interpolated image with RD ((c) and (g)) is much sharper than the bicubic interpolated one ((d) and (h)).
Partial Volume Effect

- Salvado proposed a method to restore the ideal boundary by splitting a voxel into subvoxels and reapportioning the signal into the subvoxels. Here is an example in the figure.

(a) From original Lena picture, (b) original image is interpolated with the proposed method four times.
Partial Volume Effect

- For the 2D case, let \( m \) be the original image of size \( L \times M \) voxels.

- Let \( Y = \{ y_{ij}, i = 1,2,\ldots, I, j = 1,2,\ldots, J \} \) be the new image at the higher resolution, where every voxel has been divided into \( R \times R \) voxels.

- In the 2D case, there are 2 flows to compute and distribute accordingly; horizontal and vertical.

- In both cases, neighboring set considered is the eight-connected voxels of the recipient voxel \( N_{ij} \).

\[
Q_{ij}^{\text{max}} = \frac{\text{ord}_{\{i',j'\} \in N_{ij}} \left( 6, y_{i',j'} \right) - y_{ij}}{4}
\]

\[
Q_{ij}^{\text{min}} = \frac{y_{ij} - \text{ord}_{\{i',j'\} \in N_{ij}} \left( 4, y_{i',j'} \right)}{4}
\]
Partial Volume Effect

\[ Q_{ij}^{\text{max}} = \frac{\text{ord}_{\{i, j\} \in N_{ij}}(6, y_{i', j'}) - y_{ij}}{4} \]

\[ Q_{ij}^{\text{min}} = \frac{y_{ij} - \text{ord}_{\{i, j\} \in N_{ij}}(4, y_{i', j'})}{4} \]

where \( \text{ord}_{\{i, j\}} \in (n, y_{i', j'}) \) denotes the \( n \)th highest value over the set \( N_{ij} \)

**Figure 2:** Drawing illustrating the computation of \( Q_{ij}^{\text{min}} \) and \( Q_{ij}^{\text{max}} \) for a voxel \( ij \), using rank-order statistic. Numbers show the rank of each voxel gray level in this \( 3 \times 3 \) neighborhood system; the lower the number, the smaller the intensity.
Partial Volume Effect

The equations for the horizontal and vertical flows are

\[ Q_i = \max \left[ -Q_{ij}^{\text{max}}, -Q_{i+1,j}^{\text{min}}, \min \left( Q_{i+1,j}^{\text{max}}, Q_{ij}^{\text{min}}, \hat{y}_{i+1,j}^{t} - \hat{y}_{ij}^{t} \right) \right] \]

\[ Q_j = \max \left[ -Q_{ij}^{\text{max}}, -Q_{ij+1}^{\text{min}}, \min \left( Q_{ij+1}^{\text{max}}, Q_{ij}^{\text{min}}, \hat{y}_{ij+1}^{t} - \hat{y}_{ij}^{t} \right) \right] \]

The signal will be reapportioned within an original voxel while preserving the sum over the subvoxels constant:

\[ y_{ij}^{t+1} = y_{ij}^{t} - B_{ij} \left( Q_i + Q_j \right) \]

\[ y_{i+1,j}^{t+1} = y_{i+1,j}^{t} + B_{ij} Q_i \]

\[ y_{ij+1}^{t+1} = y_{ij+1}^{t} + B_{ij} Q_j \]
Partial Volume Effect

Some results:

Correction of an actual MRI image physical phantom,

(a) The original image with 0.72 x 0.72 mm resolution.

(b) The proposed method is applied two times doubling,

(c) The proposed method is applied four time doubling,

(d) For comparison, the original interpolating four time with bicubic interpolation
Partial Volume Effect

- Some results:
Partial Volume Effect

- Students can implement this method to their own images using provided matlab source code.
Beam Hardening

- Like all x-ray beams, CT uses a polyenergetic x-ray spectrum, with energies ranging from 25 to 120 keV.
- Recall that low energy of photons cause high attenuation coefficient of tissue.
- As x-rays passes through a patient the lower energy photons will be absorbed more rapidly resulting in a higher energy of “harder” beam.
- After passing through a given thickness of patient, lower energy x-rays are attenuated to a greater extend.
Beam Hardening

**FIGURE 13-38.** The nature of x-ray beam hardening is illustrated. As a spectrum of x-rays (lower graphs) passes layers of tissue, the lower-energy photons in the x-ray spectrum are attenuated to a greater degree than the higher-energy components of the spectrum. Therefore, as the spectrum passes through increasing thickness of tissue, it becomes progressively skewed toward the higher-energy x-rays in that spectrum. In the vernacular of x-ray physics, a higher-energy spectrum is called a “harder” spectrum; hence the term beam hardening.
Beam Hardening

- Linear Attenuation Coefficient: $\mu$
  - When photon energy increases, the linear attenuation coefficient decreases.
Beam Hardening

- Therefore, as the x-ray beam propagates through a thickness of tissue and bone, the shape of spectrum becomes skewed toward the higher energies.

- Consequently, the average energy of the x-ray beam becomes greater that that of soft tissue, bone causes more beam hardening than an equivalent thickness of soft tissue.
Beam Hardening

- The beam hardening phenomenon induces artifacts in CT because rays from some projection angles are hardened to a different extent than rays from other angles, and this confuses the reconstruction algorithm.

- Solutions:
  - Most CT scanners include a simple beam-hardening correction algorithm and two-pass beam hardening correction algorithm.

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FIGURE 1.12. Beam-Hardening Artifact. A CT image of the abdomen is severely degraded by a beam-hardening artifact that produces dark streaks across the lower half of the image. The artifact was caused by marked attenuation of the x-ray beam by the patient's arms, which were kept at his sides owing to injury.

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Beam Hardening

**Fig. 4.1.** a Beam-hardening artifacts: Hounsfield bar. b Same slice as in a with an improvement using beam-hardening correction

**Fig. 4.2.** a Partial volume artifacts in the base of the skull. b Same slice as in a, but scanned with thin-collimated slices
Bias Field (Intensity Nonuniformity Field)

- Also commonly referred to as shading artifact, refers to smooth, local changes in the image intensity.
- This artifact is typically present in MRI data and depends on a combination of factors, including:
  - the shape and electromagnetic properties of the subject, or object being scanned,
  - the spatial sensitivity of the radio-frequency (RF) receiver coil,
  - gradient-driven eddy currents,
  - the frequency response of the receiver, and
  - the spatial inhomogeneity of the excitation field.
Bias Field (Intensity Nonuniformity Field)

- Correction of intensity nonuniformity is typically based on a multiplicative model of the artifact:

\[ o(x) = f(x)t(x) \]

- In this equation, \( o(x) \) and \( t(x) \) are respectively the observed and true (artifact free) signal at location \( x \), and \( f(x) \) is the distortion factor at the same position.

- Using the equation above, the true image intensities are obtained by multiplying the observed image \( o(x) \) with the reciprocal of the estimated nonuniformity field \( f(x) \).
Bias Field (Intensity Nonuniformity Field)

Figure 2.2: An MR image that exhibits intensity nonuniformity in the vertical direction (left) and the intensity profile along line A-B (right).
Bias Field (Intensity Nonuniformity Field)

- Salvado et al. proposed a method, local entropy minimization with a bicubic spline model (LEMS), to correct severe intensity inhomogeneity.

- The observed MRI signal, $Y$, is the product of the true signal, $X$, generated by the underlying anatomy and spatially varying field factor $B$ and an adaptive noise $N$. At the pixel $i$, we get

$$y_i = x_i b_i + n_i$$

- Given $Y$, the main goal is to estimate the true image $X$. But the bias field $B$ is unknown.
Bias Field (Intensity Nonuniformity Field)

- In the paper [8], the data $X$ have been generated by digitizing a sine wave such that five classes are evenly spread across the y-axis.

- (a) shows the digitized sine function, (b) shows resulting signal $Y = BX + N$ after adding the intensity inhomogeneity and the Gaussian noise, (c, bottom) show the histogram of $X$, and (c, top) shows the histogram of $Y$.
Bias Field (Intensity Nonuniformity Field)

- In the paper [8], the data $X$ have been generated by digitizing a sine wave such that five classes are evenly spread across the $y$-axis.

(Left) shows the corrected data, (right) shows histogram of the corrected data.
Bias Field (Intensity Nonuniformity Field)

One dimensional example.

(a) The original signal $X$ with five classes (values on y-axis: 20, 25, 30, 35, and 40)

(b) The signal with artifact ($Y = BX + N$).

(c) Bottom – The histogram of the data ($X$) in figure (a)

Top - The histogram of the data ($Y$) in figure (b)

(d) Corrected data

(e) Initial bias field

(f) The histogram of the corrected data in figure (d)
Salvado et al. proposed a method, local entropy minimization with a bicubic spline model (LEMS), to correct severe intensity inhomogeneity.

First, they use region growing with “regional” seeds at the top corners of the image.

From seed regions, they estimate the mean, $\mu_b$, and the standard deviation, $\sigma_b$, of the air background.

Connected pixels are included in the background if their value is less than $\mu_b + 3\sigma_b$. This process stops when no more pixels fulfill this criterion.
Bias Field (Intensity Nonuniformity Field)

- Second, a membership mask image, $M$, is created with a label of one for tissue voxels, zero for signal voids (air or flow suppressed voxels), and a value between zero and one for partial volume voxels.

- Criteria for these selections are listed below, where $\sigma_b$ is an updated estimate following region growing

$$
\begin{aligned}
&y_i < \mu_b + a\sigma_b, & m_i &= 0, \text{background} \\
&\mu_b + a\sigma_b < y_i < \mu_b + b\sigma_b & m_i &= \frac{y_i - \mu_b + a\sigma_b}{(b - a)\sigma_b} \\
&\mu_b + b\sigma_b < y_i & m_i &= 1, \text{tissue}
\end{aligned}
$$
Bias Field (Intensity Nonuniformity Field)

- When correcting the image with the estimate bias field, the signal at each voxel, $x_i$, will be corrected using

$$x_i = \frac{m_i y_i}{b_i} + (1 - m_i) y_i$$

- See [8] for more information.
Bias Field (Intensity Nonuniformity Field)

- Some results:
  - Physical Phantom

- (a) shows the test image $Y$, (b) shows the corrected image, and (c) shows the estimated Bias field.
Bias Field (Intensity Nonuniformity Field)

2-D example: Correction of physical phantom using the LEMS algorithm.

(a), (b), and (c) shows the original phantom. (d), (e), and (f) shows histograms. (g), (h), and (i) shows horizontal profiles from the middle of the images.

(a), (d), and (g) corresponds to the original phantom. (b), (e), and (h) corresponds the corrected image. (c), (f), and (i) corresponds the estimated bias field.
Bias Field (Intensity Nonuniformity Field)

- Some results:
- Clinical data

(a) shows the test image Y, (b) shows the corrected image, and (c) shows the estimated Bias field.
Bias Field (Intensity Nonuniformity Field)

2-D example: Correction of clinical data set.
(a), (b), and (c) shows Images,
(d), (e), and (f) shows histograms,
(g), (h), and (i) shows horizontal profiles from the middle of the images

(a), (d), and (g) corresponds to the original image
(b), (e), and (h) corresponds the corrected image
(c), (f), and (i) corresponds the estimated bias field
Bias Field (Intensity Nonuniformity Field)

- For more information read following paper for bias field correction.

Ring Artifact

- Ring artifact in CT occurs when a detector channel of the CT scanner does not function or when the detector does not respond uniformly.

- To keep the ring artifacts below a given level, it is necessary to maintain detector measurement accuracy within a tolerance that becomes very small for the most central detector.
Ring Artifact

- Ring artifact in CT occurs when a detectors channel of the CT scanner does not function or when the detector does not respond uniformly.

- To keep the ring artifacts below a given level, it is necessary to maintain detector measurement accuracy within a tolerance that becomes very small for the most central detector.
Motion is a common cause of artifacts encountered in any imaging study.

Motion artifacts occur when the patient moves during the acquisition. Small motions cause image blurring, and the larger physical displacements during CT image acquisition produce artifacts.
Aliasing Artifacts

- Aliasing artifact is a common artifact in MRI of the brain.

- Aliasing occurs when the field of view is smaller than the anatomic structures present in the imaged section.
Accuracy

- Image quality must be judged in the context of a specific clinical application.

- In clinical setting, we are interested in two parameters:
  - *Sensitivity*, also known as the *true-positive-fraction*; this is the fraction of patients with disease who test calls abnormal.
  - *Specificity*, also known as the *true-negative-fraction*; this is the fraction of patients without disease who test calls abnormal.
In clinical setting, we are interested in two parameters:

- **Sensitivity**, also known as the *true-positive-fraction*; this is the fraction of patients with disease who test calls abnormal.
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\[
sensitivity = \frac{a}{a + c}
\]

\[
specificity = \frac{d}{b + d}
\]

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<thead>
<tr>
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<th>Patients with disease</th>
<th>Patients without disease</th>
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<tbody>
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<td>b</td>
</tr>
<tr>
<td><strong>Test is negative</strong></td>
<td>c</td>
<td>d</td>
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Accuracy

- In other words
  - **sensitivity** = probability of a positive test among patients with disease
  - **specificity** = probability of a negative test among patients without disease

\[
sensitivity = \frac{a}{a + c}
\]

\[
specificity = \frac{d}{b + d}
\]
Accuracy

- Also

  - **Diagnostic Accuracy (DA)** is the fraction of patients that are diagnosed correctly, and is given by,

\[
DA = \frac{a + b}{a + b + c + d}
\]

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References


4. M. Bernstein, Neuro-oncology


