Solve each of the following problems on a separate sheet of paper. Write your name clearly on each one.

1. Find the output $y(t)$ of a LTI system with impulse response $h(t) = e^{-\alpha t}u(t)$ and the input $x(t) = u(t) - u(t - 5)$. Derive the Fourier Transform of $h(t)$.

2. If $x(t)$ has a Fourier Transform $X(f)$ as shown in the figure. Sketch the Fourier transform of:
   a) $ax(t)$, Where $a$ is a scalar.
   b) $x(-t)$
   c) $x(t)\cos 2\pi \beta t$
   d) $x(t) + \cos 2\pi \beta t$
   e) $x(t)\sum_{k=-\infty}^{k=\infty} \delta(t - \beta k)$.

An LTI system has an impulse response $h(t) = \frac{\sin \pi t}{\pi t}$.
For the input $x(t) = \frac{\sin \pi t}{\pi t}$, calculate:
   a) The output $y(t)$. Assume that $a < b$.
   b) $E = \int_{-\infty}^{\infty} (y(t))^2 dt$.

Using Fourier Transform properties, find the Fourier Transform of
1. $x(t) = \frac{1}{1 + (2\pi t)^2}$
2. $x(t) = \frac{\sin(2\pi t)}{\pi t}$

5. Extra Credit
   Find the value of $\alpha$, where
   $$\alpha \int_{-\infty}^{\infty} \frac{\sin(5\pi t)}{\pi t} dt + \int_{-\infty}^{\infty} \frac{\sin^2(5\pi t)}{\pi^2 t^2} dt = 10.$$