Write a Matlab code to implement the PCA and LDA methods for data reduction and classification using the following template. Consider the dataset X:

(a) Compute the sample mean
(b) Center the given dataset to have zero mean
(c) Compute the covariance matrix of the zero-mean dataset, how can you describe the given dataset by inspecting the computed covariance matrix.
(d) Compute the Eigen values of the covariance matrix.
(e) Compute the Eigen vectors of the covariance matrix.
(f) What will be the new basis of the given dataset?
(g) Project the given dataset on the new basis.
2D Case:

1. PCA

Dataset #1: Low covariance – high variance

Consider the following dataset

\[
X = \begin{pmatrix}
6 & 2 & 8 & 5 & 8 & 4 & 3 & 2 & -2 & 3 \\
12 & 19 & 4 & 10 & 8 & 6 & 7 & 6 & 9 & 3 \\
\end{pmatrix}
\]

(a) Compute the sample mean
(b) Center the given dataset to have zero mean
(c) Compute the covariance matrix of the zero-mean dataset, how can you describe the given dataset by inspecting the computed covariance matrix.
(d) Compute the Eigen values of the covariance matrix.
(e) Compute the Eigen vectors of the covariance matrix.
(f) What will be the new basis of the given dataset?
(g) Project the given dataset on the new basis.

**Dataset #2: Low covariance – low variance**

Consider the following dataset

\[
X = \begin{bmatrix}
5 & 1 & 6 & 1 & 1 & -4 & 0 & 1 & 4 & 4 \\
4 & 6 & 6 & 5 & 7 & 6 & 7 & 6 & 5 & 8 \\
\end{bmatrix}
\]

(a) Compute the sample mean
(b) Center the given dataset to have zero mean
(c) Compute the covariance matrix of the zero-mean dataset, how can you describe the given dataset by inspecting the computed covariance matrix.
(d) Compute the Eigen values of the covariance matrix.
(e) Compute the Eigen vectors of the covariance matrix.
(f) What will be the new basis of the given dataset?
(g) Project the given dataset on the new basis.

**Dataset #3: High covariance**

Consider the following dataset

\[
X = \begin{bmatrix}
1 & -8 & 4 & -5 & 1 & 6 & 1 \\
5 & -1 & 5 & 0 & 1 & 7 & 5 \\
\end{bmatrix}
\]

(a) Compute the sample mean
(b) Center the given dataset to have zero mean
(c) Compute the covariance matrix of the zero-mean dataset, how can you describe the given dataset by inspecting the computed covariance matrix.
(d) Compute the Eigen values of the covariance matrix.
(e) Compute the Eigen vectors of the covariance matrix.
(f) What will be the new basis of the given dataset?
(g) Project the given dataset on the new basis.

**2. LDA**

**Dataset #4:**

Compute the Linear Discriminant projection for the following two-dimensional dataset.
- Samples for class \(\omega_1\): \(X_1 = (x_1, x_2) = \{(4,2),(2,4),(2,3),(3,6),(4,4)\}\)
- Sample for class \(\omega_2\): \(X_2 = (x_1, x_2) = \{(9,10),(6,8),(9,5),(8,7),(10,8)\}\)

(a) compute classes mean
(b) Compute the covariance matrix of the first class
(c) Compute the covariance matrix of the second class
(d) Compute the within-class scatter matrix
(e) Compute the between-class scatter matrix
(f) Compute the LDA projection using Eigen-values and Eigen vectors
(g) Compute LDA projection directly from the scatter matrices
(h) Project the given samples (from each class) onto the computed LDA projection
(i) From the projected samples, determine a suitable threshold to be used to separate both classes from each other.

3D Case:
Consider the following dataset $X =$

1. Compute the sample mean
2. Center the given dataset to have zero mean
3. Compute the covariance matrix of the zero-mean dataset, how can you describe the given dataset by inspecting the computed covariance matrix.
4. Compute the Eigen values of the covariance matrix.
5. Compute the Eigen vectors of the covariance matrix.
6. What will be the new basis of the given dataset?
7. Project the given dataset on the new basis.

1. **PCA**

**Dataset #5: Low covariance – high variance**

Consider the following dataset

$$X = \begin{pmatrix} 0 & 0 & 0 & 1 & 3 & 7 & 4 & 2 & 6 & 10 \\ 2 & 7 & 7 & 11 & 5 & 6 & 4 & 4 & 9 & 10 \\ 9 & 5 & 12 & 12 & 8 & 4 & 9 & 4 & 6 & 6 \end{pmatrix}$$

(a) Compute the sample mean
(b) Center the given dataset to have zero mean
(c) Compute the covariance matrix of the zero-mean dataset, how can you describe the given dataset by inspecting the computed covariance matrix.
(d) Compute the Eigen values of the covariance matrix.
(e) Compute the Eigen vectors of the covariance matrix.
(f) What will be the new basis of the given dataset?
(g) Project the given dataset on the new basis.

**Dataset #6: Low covariance – low variance**

Consider the following dataset

$$X = \begin{pmatrix} 0 & 6 & -1 & -1 & 2 & 6 & 5 & -1 & -1 & -2 \\ 8 & 7 & 5 & 6 & 6 & 6 & 5 & 6 & 5 \\ 8 & 6 & 7 & 7 & 9 & 8 & 9 & 7 & 8 \end{pmatrix}$$

(a) Compute the sample mean
(b) Center the given dataset to have zero mean
(c) Compute the covariance matrix of the zero-mean dataset, how can you describe the given dataset by inspecting the computed covariance matrix.
(d) Compute the Eigen values of the covariance matrix.
(e) Compute the Eigen vectors of the covariance matrix.
(f) What will be the new basis of the given dataset?
(g) Project the given dataset on the new basis.

Dataset #7: High covariance

Consider the following dataset

\[
X = \begin{pmatrix}
1 & 0 & 7 & 4 & 1 & 2 & 10 & 2 & 2 & 6 \\
5 & 5 & 10 & 9 & 4 & 6 & 13 & 6 & 6 & 9 \\
8 & 6 & 11 & 8 & 9 & 14 & 8 & 9 & 12
\end{pmatrix}
\]

(a) Compute the sample mean
(b) Center the given dataset to have zero mean
(c) Compute the covariance matrix of the zero-mean dataset, how can you describe the given dataset by inspecting the computed covariance matrix.
(d) Compute the Eigen values of the covariance matrix.
(e) Compute the Eigen vectors of the covariance matrix.
(f) What will be the new basis of the given dataset?
(g) Project the given dataset on the new basis.

2. LDA

Dataset #8:

Compute the Linear Discriminant projection for the following two-dimensional dataset.

Samples for class $\omega_1$:

\[
\begin{array}{cccccccccc}
0 & 6 & 4 & 3 & 4 & 2 & 1 & 2 & 3 & 5 \\
3 & 9 & 3 & 8 & 8 & 5 & 3 & 4 & 6 & 10 \\
9 & 12 & 1 & 7 & 9 & 10 & 5 & 11 & 10 & 8
\end{array}
\]

Sample for class $\omega_2$:

\[
\begin{array}{cccccccccc}
6 & 4 & 7 & 1 & 3 & 4 & 5 & 9 & 6 & 0 \\
4 & 13 & 6 & 4 & 5 & 6 & 10 & 9 & 10 & 2 \\
5 & 13 & 6 & 14 & 4 & 5 & 7 & 11 & 6 & 9
\end{array}
\]

(a) Compute classes mean
(b) Compute the covariance matrix of the first class
(c) Compute the covariance matrix of the second class
(d) Compute the within-class scatter matrix
(e) Compute the between-class scatter matrix
(f) Compute the LDA projection using Eigen-values and Eigen vectors
(g) Project the given samples (from each class) onto the computed LDA projection