1. Let \( \{X_i\}, i = 1, 2, \ldots, n \) be iid Rayleigh;

\[ f_X(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right); \quad x \geq 0. \]

Let \( S = \max \{ X_i \}, i = 1, 2, \ldots, n \)

\[ E(S^2) = 2 \sigma^2 \sum_{k=1}^{n} k. \]

**Proof**

First, let's get the density function of the RV \( S \).

\[ F_S(s) = P(S \leq s) \]

\[ = P(X_1 \leq s, X_2 \leq s, \ldots, X_n \leq s) \]

\[ = P(X_1 \leq s) \cdot P(X_2 \leq s) \ldots P(X_n \leq s) \]

\[ \Rightarrow F_S(s) = \left( F_X(s) \right)^n; \quad X_i \text{ are iid} \quad (1) \]

Differentiating w.r.t. \( s \), we get

\[ f_S(s) = n \left( F_X(s) \right)^{n-1} f_X(s). \quad (2) \]
Now, \( F_X(x) \triangleq P(X \leq x) \)

\[
= \int_{-\infty}^{x} f_X(x') \, dx' = \int_{-\infty}^{x} f_X(x) \, dx, \quad x \geq 0
\]

\[
= \int_{0}^{x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2\sigma^2} \, dx
\]

\[
F_X(x) = 1 - e^{-x^2/2\sigma^2} \tag{3}
\]

\[
F_X(s) = 1 - e^{-s^2/2\sigma^2} \tag{4}
\]

Hence,

\[
f_S(s) = n \left[ 1 - e^{-s^2/2\sigma^2} \right] \frac{s}{\sigma^2} \tag{5}
\]

Now,

\[
E(s^2) \triangleq \int_{-\infty}^{\infty} s^2 f_S(s) \, ds
\]

\[
= \int_{0}^{\infty} \frac{n s^3}{\sigma^2} \left[ 1 - e^{-s^2/2\sigma^2} \right] \frac{s}{\sigma^2} \, ds
\]

\[
= \sum_{i=0}^{n-1} (n-1) \cdot (1 - e^{-s^2/2\sigma^2}) \tag{6}
\]

To simplify the integral, we note that

\[
(1 - e^{-s^2/2\sigma^2})^{-1} = \sum_{i=0}^{n-1} (n-1) \cdot (1 - e^{-s^2/2\sigma^2}) \tag{7}
\]
Therefore,

\[
E(s^2) = \frac{n}{\sigma^2} \sum_{r=0}^{n-1} \binom{n-1}{r} (-1)^r \int_0^\infty s^2 e^{-s^2/2\sigma^2} (r+1)^{-2} \, ds
\]

\[
= \frac{n}{\sigma^2} \sum_{r=0}^{n-1} \binom{n-1}{r} (-1)^r \frac{2\sigma^4}{(r+1)^2}
\]

\[
= 2\sigma^2 \sum_{J=1}^n \binom{n-1}{J-1} (-1)^{J-1} \frac{1}{J^2}
\]

\[
= 2\sigma^2 \sum_{J=1}^n \frac{(-1)^{J+1} \binom{n}{J}}{J}
\]

\[
= 2\sigma^2 \sum_{k=1}^{n} \frac{1}{k}
\]

From Tables of integrals, series and products:
2. \( X_n \xrightarrow{p} X \) if and only if \( \lim_{n \to \infty} E \left( \frac{|X_n - X|}{1 + |X_n - X|} \right) = 0 \)

**Proof:**

Let \( X_n - X = Y_n \).

\[
E \left( \frac{|Y_n|}{1 + |Y_n|} \right) = \int_{-\infty}^{\infty} \frac{|y_n|}{1 + |y_n|} f_{Y_n}(y_n) \, dy_n
\]

1. \( = \int_{|y_n| \geq \varepsilon} \frac{|y_n|}{1 + |y_n|} f_{Y_n}(y_n) \, dy_n + \int_{|y_n| < \varepsilon} \frac{|y_n|}{1 + |y_n|} f_{Y_n}(y_n) \, dy_n \)

2. \( \geq \int_{|y_n| \geq \varepsilon} \frac{\varepsilon}{1 + \varepsilon} f_{Y_n}(y_n) \, dy_n \)

\[
\geq \frac{\varepsilon}{1 + \varepsilon} \int_{|y_n| \geq \varepsilon} f_{Y_n}(y_n) \, dy_n
\]

3. \( \geq \frac{\varepsilon}{1 + \varepsilon} \cdot P(|Y_n| \geq \varepsilon) \)

4. \[
E \left( \frac{|Y_n|}{1 + |Y_n|} \right) \geq \frac{\varepsilon}{1 + \varepsilon} \cdot P(|Y_n| \geq \varepsilon)
\]
Also, from (3)
\[ E\left( \frac{|Y_n|}{1+|Y_n|} \right) \leq \frac{3}{1+3} \int_{|Y_n| \geq 3} f_{Y_n}(y_n) \, dy_n + \frac{3}{1+3} \int_{|Y_n| < 3} f_{Y_n}(y_n) \, dy_n. \] 
(check by choosing some values of \( n \in \mathbb{N} \).)

\[ \leq P(|Y_n| \geq 3) + \frac{3}{1+3} P(|Y_n| < 3), \]
\[ \text{for all } \varepsilon > 0 \]

\[ \therefore \left\{ \begin{align*}
E\left( \frac{|Y_n|}{1+|Y_n|} \right) &\leq P(|Y_n| \geq 3) + \frac{3}{1+3} P(|Y_n| < 3) \quad (6) \\
&\leq P(|Y_n| \geq \varepsilon) + \frac{3}{1+3} \varepsilon \quad (9)
\end{align*} \right. \]

Equations 6 and 10 are the lower and upper bounds of the quantity in question.

Now let \( \lim_{n \to \infty} E\left( \frac{|Y_n|}{1+|Y_n|} \right) = 0 \) (i.e., assume that \( X_n \xrightarrow{m.s.} X \)).

Then, the RHS of (6) \( \to 0 \) as \( n \to \infty \);

i.e., \( \lim_{n \to \infty} P(|Y_n| \geq \varepsilon) = P(|X_n - X| \geq \varepsilon) = 0 \)
\[ \Rightarrow X_n \xrightarrow{P} X. \]
Now let \( \lim_{n \to \infty} P(|Y_n| \geq \varepsilon) = 0 \) (i.e., assume that \( X_n \xrightarrow{p} X \).

From 10 by taking the limits of the two sides,

\[
\lim_{n \to \infty} \mathbb{E}\left( \frac{|Y_n|}{1+|Y_n|} \right) \leq \frac{\varepsilon}{1+\varepsilon} \quad \text{\( \therefore \)}
\]

But \( \varepsilon \) is an arbitrary quantity which we can select. Therefore, we can select \( \varepsilon \) to be very small number such that

\[
\lim_{n \to \infty} \mathbb{E}\left( \frac{|Y_n|}{1+|Y_n|} \right) \to 0.
\]

\[
\Rightarrow \lim_{n \to \infty} \mathbb{E}(|Y_n|) \to 0
\]

\( \text{i.e., } \lim_{n \to \infty} \mathbb{E}(|X_n - X|) \to 0 \)

\[
\Rightarrow X_n \xrightarrow{m.s} X.
\]

\( \text{Q.E.D.} \)
3. Given $z_x = z_y = z_z = 0$
$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$.

$$r_xz = r_x y r_z - \sqrt{1 - r_x^2} \cdot \sqrt{1 - r_y^2} = r_y z$$

**Proof**

From notes, pp. 11-13, we stated that the correlation matrix $R$ is nonnegative definite, i.e.

$$\begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix} > 0,$$  \hspace{1cm} (1)

where $R_{ij} = E(X_i \cdot X_j)$

$$= C_{ij} \quad \text{since } z_i = 0 \quad \text{in our case}$$

$$\Delta = r_{ij} \cdot \sigma_i \cdot \sigma_j$$ \hspace{1cm} (2)

$$R = \begin{bmatrix} r_{xx} \sigma_x^2 & r_{xy} \sigma_x \sigma_y & r_{xz} \sigma_x \sigma_z \\ r_{yx} \sigma_y \sigma_x & r_{yy} \sigma_y^2 & r_{yz} \sigma_y \sigma_z \\ r_{zx} \sigma_z \sigma_x & r_{zy} \sigma_z \sigma_y & r_{zz} \sigma_z^2 \end{bmatrix}$$ \hspace{1cm} (3)
\[
R = \begin{bmatrix}
1 & r_{xy} & r_{xz} \\
r_{xy} & 1 & r_{yz} \\
r_{xz} & r_{yz} & 1
\end{bmatrix}
\] 

\[\text{Note: } r_{xy} = r_{yy} = r_{zz} = 1\]

and, it's given that \( \sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1 \).

Now since the determinant of \( R \) is \( > 0 \)

\[ (1 - r_{yz}^2) - r_{xy} (r_{xy} - r_{yz} r_{xz}) + r_{xz} (r_{xy} r_{yz} - r_{xz}) \geq 0 \]

\[ 1 - r_{yz}^2 - r_{xy} + 2 r_{xy} r_{yz} r_{xz} > r_{xz}^2 \]

Adding \( r_{xy}^2 \) and \( r_{yz}^2 \) to both sides

\[ (1 - r_{xy}^2) (1 - r_{yz}^2) \geq (r_{xy} r_{yz} - r_{xz})^2 \]

Hence, \[ r_{xy} r_{yz} - r_{xz} \leq \sqrt{1 - r_{xy}^2} \cdot \sqrt{1 - r_{yz}^2} \]

\[ \Rightarrow \quad r_{xz} \geq r_{xy} r_{yz} - \sqrt{1 - r_{xy}^2 \cdot 1 - r_{yz}^2} \]
4. Let the distance in pace i be $d_i$. Hence, the total distance in 100 paces will be
$$ d = \sum_{i=1}^{100} d_i, \quad \text{(1)} $$
where $d_i$ is i.i.d. with mean 0.97 and variance 0.01. Therefore,
$$ \bar{d} = 100 \bar{d} = 97 $$
$$ \sigma_d^2 = 100 \sigma^2 = 1 $$

Recall: If $Y = \sum_{i=1}^{N} x_i$ and $x_i$ are i.i.d with mean $\mu_x$ and variance $\sigma_x^2$,
then
$$ \bar{Y} = 100 \mu_x $$
$$ \sigma_Y^2 = 100 \sigma_x^2. \quad (\text{shw!}) $$

Now, we need to calculate
$$ P(95 < d < 105). $$
$$ P(95 < d < 105) = P \left( \frac{95 - 2d}{\sigma_d} < \frac{d - \mu_d}{\sigma_d} < \frac{105 - 2d}{\sigma_d} \right) $$
\[ P \left( -2 < d_n < 8 \right), \quad d_n \triangleq \frac{d - 2d}{\sigma_d} \]

Now, using the central limit theorem approximations, we can approximate the above probability as a Gaussian distribution, 
that is,
\[ d_n \triangleq \frac{d - 2d}{\sigma_d} \sim N \left( 0, 1 \right) \]

Hence,
\[ P \left( -2 < d_n < 8 \right) = G(8) - G(-2) \]
\[ = G(8) - (1 - G(2)) \]
\[ = G(8) + G(2) - 1 \]
\[ \approx G(2) \]
\[ = 0.9772 \]

Note: From the given table
\[ G(x) \triangleq 1 \quad \text{for } x \geq 4 \]
\[ G(y) \triangleq 0 \quad \text{for } y \leq -4 \]

So, we took \[ G(8) \approx 1 \].
Table I  Values of the standard normal distribution function

\[
\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du = P(Z \leq z)
\]

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Table I  Values of the standard normal distribution function

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Note 1: If a normal variable $X$ is not "standard," its values must be "standardized": $Z = (X - \mu)/\sigma$. That is, $P(X \leq z) = \Phi \left( \frac{z - \mu}{\sigma} \right)$.

Note 2: For "two-tail" probabilities, see Table Ia.

Note 3: For $z \geq 4$, $\Phi(z) = 1$ to four decimal places; for $z \leq -4$, $\Phi(z) = 0$ to four decimal places.

Note 4: The entries opposite $z = 3$ are for $z = 3.0, 3.1, 3.2, \text{etc.}$.