ECE 530 Homework # 3 (Issued Thursday 1/31, Due Thursday 2/7)

1. Prove that, in general, the union of two $\sigma$–algebras is not a $\sigma$–algebra, while the intersection of two $\sigma$–algebras is a $\sigma$–algebra.

2. Text # 2-23 pp.45

3. Text # 2-25 pp.45

4. Text # 3-4 pp. 71

5. Text # 4-26 pp. 121

6. Consider a game which consists of two successive trials; the first trial has outcomes A and B and the second has outcomes C and D. The probabilities of the four possible outcomes of the game are as follows:

<table>
<thead>
<tr>
<th>Outcome</th>
<th>AC</th>
<th>AD</th>
<th>BC</th>
<th>BD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>1/3</td>
<td>1/6</td>
<td>1/6</td>
<td>1/3.</td>
</tr>
</tbody>
</table>

Prove/disprove that the events A and C are statistically independent.

7. You are one of $n > 1$ people. A special task is to be assigned to the person who draws the short straw from a set of $n$ straws. What place inline should you take to maximize your chance of you not being chosen? Prove your answer. (Hint: Derive an expression for the probability of drawing the short straw as a function of your position in the line.)