**Image Enhancement: Frequency domain methods**

- The concept of filtering is easier to visualize in the frequency domain. Therefore, enhancement of image \( f(m,n) \) can be done in the frequency domain, based on its DFT \( F(u,v) \).

- This is particularly useful, if the spatial extent of the point-spread sequence \( h(m,n) \) is large. In this case, the convolution

\[
g(m,n) = h(m,n) \ast f(m,n)
\]

Enhanced Image  
Given Image

may be computationally unattractive.

- We can therefore directly design a transfer function \( H(u,v) \) and implement the enhancement in the frequency domain as follows:

\[
G(u,v) = H(u,v)F(u,v)
\]

Enhanced Image  
Given Image
Lowpass filtering

- Edges and sharp transitions in grayvalues in an image contribute significantly to high-frequency content of its Fourier transform.

- Regions of relatively uniform grayvalues in an image contribute to low-frequency content of its Fourier transform.

- Hence, an image can be smoothed in the Frequency domain by attenuating the high-frequency content of its Fourier transform. This would be a lowpass filter!

- For simplicity, we will consider only those filters that are real and radially symmetric.

- An ideal lowpass filter with cutoff frequency $r_0$:

$$H(u,v) = \begin{cases} 
1, & \text{if } \sqrt{u^2 + v^2} \leq r_0 \\
0, & \text{if } \sqrt{u^2 + v^2} > r_0
\end{cases}$$
• Note that the origin (0, 0) is at the center and not the corner of the image (recall the “fftshift” operation).

• The abrupt transition from 1 to 0 of the transfer function $H(u,v)$ cannot be realized in practice, using electronic components. However, it can be simulated on a computer.
Ideal LPF examples

Original Image

LPF image, $r_0 = 57$

LPF image, $r_0 = 36$

LPF image, $r_0 = 26$

- Notice the severe ringing effect in the blurred images, which is a characteristic of ideal filters. It is due to the discontinuity in the filter transfer function.
Choice of cutoff frequency in ideal LPF

- The cutoff frequency $r_0$ of the ideal LPF determines the amount of frequency components passed by the filter.
- Smaller the value of $r_0$, more the number of image components eliminated by the filter.
- In general, the value of $r_0$ is chosen such that most components of interest are passed through, while most components not of interest are eliminated.
- Usually, this is a set of conflicting requirements. We will see some details of this is image restoration.
- A useful way to establish a set of standard cut-off frequencies is to compute circles which enclose a specified fraction of the total image power.
  - Suppose $P_T = \sum_{v=0}^{N-1} \sum_{u=0}^{M-1} P(u,v)$, where $P(u,v) = |F(u,v)|^2$, is the total image power.
- Consider a circle of radius $r_0(\alpha)$ as a cutoff frequency with respect to a threshold $\alpha$ such that $\sum_{v} \sum_{u} P(u,v) = \alpha P_T$.
- We can then fix a threshold $\alpha$ and obtain an appropriate cutoff frequency $r_0(\alpha)$. 
Butterworth lowpass filter

- A two-dimensional Butterworth lowpass filter has transfer function:

\[
H(u, v) = \frac{1}{1 + \left[\frac{\sqrt{u^2 + v^2}}{r_0}\right]^{2n}}
\]

- \( n \): filter order, \( r_0 \): cutoff frequency

- Frequency response does not have a sharp transition as in the ideal LPF.

- This is more appropriate for image smoothing than the ideal LPF, since this not introduce ringing.
Butterworth LPF example

Original Image

LPF image, $r_0 = 18$

LPF image, $r_0 = 13$

LPF image, $r_0 = 10$
Butterworth LPF example: False contouring

Image with false contouring due to insufficient bits used for quantization

Lowpass filtered version of previous image
Butterworth LPF example: Noise filtering

Original Image

Noisy Image

LPF Image
Gaussian Low pass filters

- The form of a Gaussian lowpass filter in two-dimensions is given by \( H(u, v) = e^{-D^2(u,v)/2\sigma^2} \), where \( D(u, v) = \sqrt{u^2 + v^2} \) is the distance from the origin in the frequency plane.

- The parameter \( \sigma \) measures the spread or dispersion of the Gaussian curve. Larger the value of \( \sigma \), larger the cutoff frequency and milder the filtering.

- When \( D(u, v) = \sigma \), the filter is down to 0.607 of its maximum value of 1.

- See Example 4.6 in the text for an illustration.

- Also read section 4.3.4 for an application of lowpass filtering to text images.
Highpass filtering

- Edges and sharp transitions in grayvalues in an image contribute significantly to high-frequency content of its Fourier transform.

- Regions of relatively uniform grayvalues in an image contribute to low-frequency content of its Fourier transform.

- Hence, image sharpening in the Frequency domain can be done by attenuating the low-frequency content of its Fourier transform. This would be a highpass filter!

- For simplicity, we will consider only those filters that are real and radially symmetric.

- An ideal highpass filter with cutoff frequency $r_0$:

$$H(u,v) = \begin{cases} 
0, & \text{if } \sqrt{u^2 + v^2} \leq r_0 \\
1, & \text{if } \sqrt{u^2 + v^2} > r_0
\end{cases}$$
• Note that the origin (0, 0) is at the center and not the corner of the image (recall the “fft shift” operation).

• The abrupt transition from 1 to 0 of the transfer function \( H(u, v) \) cannot be realized in practice, using electronic components. However, it can be simulated on a computer.
Ideal HPF examples

- Notice the severe ringing effect in the output images, which is a characteristic of ideal filters. It is due to the discontinuity in the filter transfer function.
**Butterworth highpass filter**

- A two-dimensional Butterworth highpass filter has transfer function:

\[
H(u, v) = \frac{1}{1 + \left( \frac{r_0}{\sqrt{u^2 + v^2}} \right)^{2n}}
\]

- \(n\): filter order, \(r_0\): cutoff frequency

- Frequency response does not have a sharp transition as in the ideal HPF.

- This is more appropriate for image sharpening than the ideal HPF, since this does not introduce ringing.

**Butterworth HPF with**

\(r_0 = 47\) and 2
Butterworth HPF example

Original Image

HPF image, $r_0 = 47$

HPF image, $r_0 = 36$

HPF image, $r_0 = 81$
Gaussian High pass filters

- The form of a Gaussian lowpass filter in two-dimensions is given by $H(u, v) = 1 - e^{-D^2(u,v)/2\sigma^2}$, where $D(u, v) = \sqrt{u^2 + v^2}$ is the distance from the origin in the frequency plane.
- The parameter $\sigma$ measures the spread or dispersion of the Gaussian curve. Larger the value of $\sigma$, larger the cutoff frequency and more severe the filtering.
- See Example in section 4.4.3 of text for an illustration.