(a) \( A \Theta B^4 \)

\[
\begin{array}{c}
(A \Theta B_4) \Theta B^2
\end{array}
\]

\[
\begin{array}{c}
(A \Theta B_4) \Theta B^2
\end{array}
\]
(b) \((A \oplus B^1) \oplus B^3\)

\[\begin{align*}
A \oplus B^1 \\
(A \oplus B^1) \oplus B^3
\end{align*}\]
(d) \((A \oplus B^3) \ominus B^2\)
element. Solution (d) was obtained by first dilating the original set with the large disk shown. Then dilated image was then eroded with a disk of half the diameter of the disk used for dilation.

![Image of dilated and eroded set](image)

Figure P9.5

Problem 9.8

(a) The dilated image will grow without bound. (b) A one-element set (i.e., a one-pixel image).

Problem 9.10

The approach is to prove that

\[
\{ x \in \mathbb{Z}^2 \mid (\mathring{B})_x \cap A \neq \emptyset \} \equiv \{ x \in \mathbb{Z}^2 \mid x = a + b \text{ for } a \in A \text{ and } b \in B. \}
\]

The elements of \((\mathring{B})_x\) are of the form \(x - b\) for \(b \in B\). The condition \((\mathring{B})_x \cap A \neq \emptyset\) implies that for some \(b \in B\), \(x - b \in A\), or \(x - b = a\) for some \(a \in A\) (note in the preceding equation that \(x = a + b\)). Conversely, if \(x = a + b\) for some \(a \in A\) and \(b \in B\), then \(x - b = a\) or \(x - b \in A\), which implies that \((\mathring{B})_x \cap A \neq \emptyset\).

Problem 9.12

The proof, which consists of proving that

\[
\{ x \in \mathbb{Z}^2 \mid x + b \in A, \text{ for every } b \in B \} \equiv \{ x \in \mathbb{Z}^2 \mid (B)_x \subseteq A \},
\]

follows directly from the definition of translation because the set \((B)_x\) has elements of the form \(x + b\) for \(b \in B\). That is, \(x + b \in A\) for every \(b \in B\) implies that \((B)_x \subseteq A\). Conversely, \((B)_x \subseteq A\) implies that all elements of \((B)_x\) are contained in \(A\), or \(x + b \in A\) for every \(b \in B\).