NON-METRIC CALIBRATION OF CAMERA LENS DISTORTION

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ABSTRACT

This paper addresses the problem of calibrating camera lens distortion, which can be significant in medium to wide angle lenses. Our approach is based on the analysis of distorted images of straight lines. We derive a new distortion measure that can be optimized using non-linear search techniques to find the best distortion parameters that straighten these lines. Unlike the other existing approaches, we also show how to use this measure to find fast, closed-form solutions to the distortion coefficients. Some experiments to evaluate the performance of this approach on synthetic and real data are reported.

1. INTRODUCTION

Calibration is an important component of any vision task which seeks to extract geometric information from a scene. Modeling the imaging device as an ideal pinhole camera, calibration requires the determination of 11 parameters of a camera including its position and orientation in space (extrinsic parameters), the image center, scale factor and lens focal length (intrinsic parameters). Being linear if expressed in terms of projective geometry, the pinhole model simplifies a lot of considerations on geometry in which cameras are involved. However, for some applications which require high accuracy, or in cases where low-cost or wide-angle lens are used, the pinhole model is not sufficient and more parameters should be estimated to take into account camera lens distortion.

The distortion parameters are most often estimated along with all (extrinsic and intrinsic) parameters of the camera model (see for example [1, 2]). This is done using a set of 3D-to-2D correspondences extracted with the help of a calibration object of known structure. The problem with these methods is the fact that there is some kind of coupling between internal parameters, including distortion parameters, and external parameters that result in high errors on the camera internal parameters [3]. Moreover obtaining accurate coordinates of 3D scene points is sometimes demanding or impossible (e.g., in case of snapshots already recorded).

In contrast, another family of non-metric methods have been proposed, which do not rely on known scene points [4, 3, 5, 6]. Instead, these methods rely on the fact that straight lines in the scene must always perspective project to straight lines in the image. This means that curvature of lines in the image is due to lens distortion. Using this principle, distortion parameters that map distorted image curves to straight lines can be estimated. Once estimated, the images can be undistorted by applying the inverse of the distortion function to the entire image or image features, thus allowing the camera to be considered as an ideal pinhole camera. These methods therefore pave the way for many calibration techniques such as calibration from vanishing points and auto-calibration methods, which rely on a pinhole model or do not provide any method of accounting for lens distortion.

In this paper, we propose a non-metric approach to camera lens calibration. We derive a new distortion measure that can be optimized using non-linear search techniques to find the best distortion parameters that straighten the image lines. Unlike the other existing approaches, we also show how to use this measure to find fast, closed-form solutions to the distortion coefficients. This paper is organized as follows. Section II describes the camera distortion model. Section III presents our method to straighten the distorted image lines and its closed-form solutions. Some experimental results are reported in Section IV, followed by our concluding remarks in Section V.

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2. CAMERA DISTORTION MODEL

Lens distortion parameters are inherently different from the other camera parameters, since they model a point to point mapping on the image plane and require no specific knowledge of the 3D scene which generated the image points. The two principal forms of distortion considered in videometry and photogrammetry applications are radial and decentering distortion. The standard model for the radial and decentering distortion [7] is mapping from the distorted image coordinates, \((x_d, y_d)\), to the undistorted image plane coordinates, \((x_u, y_u)\), which are not physically measurable, according to the equation:

\[
x_u = x_d + \overline{x_d}(K_1 r_d^2 + K_2 r_d^4 + K_3 r_d^6 + \ldots) + [P_1(r_d^2 + 2\overline{x_d}) + 2P_2 \overline{x_d} y_d][1 + P_3 r_d^2 + \ldots] + \nonumber\]

\[
y_u = y_d + \overline{y_d}(K_1 r_d^2 + K_2 r_d^4 + K_3 r_d^6 + \ldots) + [P_2(r_d^2 + 2\overline{y_d}) + 2P_1 \overline{x_d} y_d][1 + P_3 r_d^2 + \ldots], \tag{1}\]

where

\[
x_d = x_d - c_x, \quad y_d = y_d - c_y, \quad r_d^2 = x_d^2 + y_d^2,
\]

and \(K_1, K_2, K_3\) are the coefficients of radial distortion and \(P_1, P_2\) and \(P_3\) are the coefficients of the decentering distortion. \(r_d\) is the radius of an image point from the distortion center, defined as \((c_x, c_y)\) above. Typically, only one or two distortion parameters are modeled, as the higher order terms are comparatively insignificant [2]. Moreover, for many machine vision applications, the decentering distortion component need not to be considered [1]. The lens distortion calibration problem thus becomes to recover the practically significant distortion coefficients along with the distortion center \((c_x, c_y)\).

3. LINE STRAIGHTNESS METHOD

The goal of the distortion calibration is to find the transformation that maps the actual camera image plane onto an image following the perspective camera model. To find the distortion parameters, the following fundamental property is often used: a camera follows the perspective camera model if and only if the projection of every 3D line in space onto the camera plane is a line. Consequently, all one needs is a way to find projections of 3D lines in the image, and a way to measure how much each line is distorted in the image. This distortion measure will then be minimized to find the best calibration parameters. One common such measure is the sum of squared distances of the edge points from the straight lines on which they should lie [4, 3, 5]. Other similar distortion measures could be used as well, e.g., the mean curvature of the line points. These measures lead to non-linear objective functions that need efficient search algorithms. In what follows, we derive a new distortion measure that can be minimized by non-linear optimization algorithms and from which, however, closed-form solutions can also be derived to solve for the distortion parameters.

3.1. Proposed Distortion Measure

Our proposed measure follows the fundamental property stated above. Suppose we have a line \(l\) in the undistorted image plane. Every point \((x_u, y_u)\) on the line satisfies the equation

\[a x_u + b y_u + c = 0, \tag{2}\]

where \(a\), \(b\) and \(c\) are constants for the specific line \(l\), with \(s = -a/b\) is the line slope. Each point on the line is related to a point \((x_d, y_d)\) in the distorted image plane according to (1). This means that both coordinates of the line point are functions of \((x_d, y_d)\). Accordingly, the last equation can be written as

\[f(x_d, y_d) = a x_u(x_d, y_d) + b y_u(x_d, y_d) + c = 0, \tag{3}\]

where \(f(x_d, y_d)\) describes the equation of the corresponding curve in the distorted image plane. The elemental change in \(f\) at any distorted image point \((x_d, y_d)\) can be expressed as

\[
\delta f = a \frac{\partial x_u}{\partial x_d} \delta x_d + \frac{\partial x_u}{\partial y_d} \delta y_d + b \frac{\partial y_u}{\partial x_d} \delta x_d + \frac{\partial y_u}{\partial y_d} \delta y_d = 0, \tag{4}\]

where all the four partial derivatives can be directly computed from (1). Hence, one can see that the slope of the line, \(s\), in the undistorted plane is related to the slope of the tangent, \(\frac{dy_u}{dx_u}\), to the curve at point \((x_d, y_d)\) by

\[s(x_d, y_d) = \frac{\frac{\partial y_u}{\partial x_u} + \frac{\partial y_u}{\partial y_d} \frac{\delta y_d}{\delta x_d}}{\frac{\partial x_u}{\partial x_d} + \frac{\partial y_u}{\partial y_d} \frac{\delta y_d}{\delta x_d}}. \tag{5}\]

In the problem of distortion calibration, we usually have a number of distorted points in the image plane. Under the correct values of the distortion parameters, the slopes computed from the last equation for all these points should be the same if the points are to lie on the same line in the undistorted image. Therefore, we can define the following distortion measure. Given a chain of edge points, \((x_d^i, y_d^i), i = 1, \ldots, N\), that should belong to the same line in the undistorted image, we can compute approximately the slopes of the tangents at
the chain points (e.g., using first difference or any more robust method such as function fitting), and hence we can solve for the distortion parameters that minimize the error

$$E = \sum_{i=2}^{N} (s(x_d^i, y_d^i) - s(x_d^{i-1}, y_d^{i-1}))^2. \quad (6)$$

Clearly this measure would be zero if the points are mapped to a perfect linear segment in the undistorted image and the more the segment would be distorted, the bigger the measure. For many lines, the sum of the error in (6) for all the lines can be used.

3.1.1. Closed-form Solution

The distortion measure in (6) can be minimized using non-linear optimization algorithms starting from an appropriate guess of the distortion parameters (e.g., $(c_x, c_y)$) initially at the center of the image). However, closed-form solutions for rest of the distortion parameters can be obtained if a good estimate of the distortion center is known. The distortion center can be independently estimated with good accuracy using some other existing techniques such as the autocollimated laser technique [8]. This situation is also applicable if solving for the distortion coefficients is nested within a coarse-to-fine search for the distortion center (as the case in [6]). In this case, the closed-form solution will provide a fast, one-shot solution to the distortion coefficients.

The reason behind the existence of closed-form solutions comes from the fact that our distortion measure in (6) will be linear in the distortion coefficients, once the distortion center is known. This is not the case with the distortion measures used by other researchers. For example if we assume, without loss of generality, that we want to estimate only $K_1$ and $P_1$ from the distortion coefficients. The partial derivatives used in (5) can be easily found to be

$$\frac{\partial y_u}{\partial x_d} = 2K_1(x_d - c_x)(y_d - c_y) + 2P_1(y_d - c_y),$$

$$\frac{\partial y_u}{\partial y_d} = 1 + 3K_1(y_d - c_y)^2 + K_1(x_d - c_x)^2 + 2P_1(x_d - c_x),$$

$$\frac{\partial x_u}{\partial x_d} = 1 + 3K_1(x_d - c_x)^2 + K_1(y_d - c_y)^2 + 6P_1(x_d - c_x),$$

$$\frac{\partial x_u}{\partial y_d} = 2K_1(x_d - c_x)(y_d - c_y) + 2P_1(y_d - c_y).$$

All these quantities are linear in $K_1$ and $P_1$ at any given point $(x_d, y_d)$. The slope, $s$, that all the same-line points should have can be estimated from the points by least-square linear regression. Each point will yield a linear equation and all of them can be stacked in the form $A X = B$, where $A$ is a $N \times M$ matrix, with $N$ is the number of points used for each line and $M$ is the number of lines, $X = [K_1 P_1]^T$ and $B$ is a vector of length $N \times M$. This over-determined set of equations can be efficiently solved using singular value decomposition. To improve our first estimates of the lines slopes, they could be re-computed from the points corrected by the obtained distortion coefficients, then a new, more accurate set of the coefficients is estimated again in the same way.

It is important to note that if higher accuracy is required, the closed form solution can be taken as starting point of a non-linear optimization algorithm that minimizes the error in (6).

4. EXPERIMENTAL RESULTS

In this section, the performance of our technique is assessed using both synthetic and real image data. The synthetic images provide exact knowledge of line positions, orientations and distortion parameters, so precise quantitative evaluation of performance is possible. The performance on real images is shown to demonstrate the practical implementation of the technique.

4.1. Synthetic Images

A $320 \times 242$ image consisting of 10 lines is used a test image. The lines were generated with random orientations and positions. Using known distortion parameters, the line points were distorted, see Fig. 1(a). To simulate errors in feature extraction, a zero-mean Gaussian noise with standard deviation 0.1 pixels was added. We then used our approach to estimate the distortion parameters from the noisy data and used these parameters to undistort the image, see Fig. 1(b). We approximated the slope of the tangent to the curve at a point by central differences averaged over a window of size 5 around the point. A more robust method to estimate the tangent can improve the results. Table 1 shows the distortion parameters used to create the image and the results obtained. The second row shows the results of the closed-form solution, followed in the third row by the results of the optimization of the distortion measure in (6) using a modified version of the Levenberg-Marquardt algorithm (LM) available from the software package MINPACK. The run time in seconds on an SGI-O2 machine for the linear approach is 0.20, while it is 3.67 seconds for the non-linear approach. Clearly, the linear solution provides a very
fast response but the non-linear optimization algorithm provides more accurate results.

![Fig. 1. Performance on synthetic images: (a) input distorted, noisy image. (b) output undistorted image.](image)

Table 1. Results of closed-form and non-linear solutions on synthetic images.

<table>
<thead>
<tr>
<th>Method</th>
<th>$c_x$</th>
<th>$c_y$</th>
<th>$K_1$</th>
<th>$P_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>130</td>
<td>140</td>
<td>$20\times10^{-8}$</td>
<td>$-3\times10^{-7}$</td>
</tr>
<tr>
<td>closed</td>
<td>given</td>
<td>given</td>
<td>$1.635\times10^3$</td>
<td>$-2.60\times10^{-4}$</td>
</tr>
<tr>
<td>nonlin.</td>
<td>128.29</td>
<td>139.39</td>
<td>$2.06\times10^{-4}$</td>
<td>$-2.93\times10^{-4}$</td>
</tr>
</tbody>
</table>

4.2. Real Images

Our approach is also applied to real images acquired by an IndyCam in our lab. Since image distortion is sometimes less than a pixel, we used an edge detection method with a sub-pixel accuracy [9], which is quite robust to noise. We used a threshold of about 40 on the length of the resulting edge chains because small segments may contain more noise than useful information about distortion. Moreover, because of the corner rounding effect [3] due to edge detection, about 5 edge points at both ends of each chain are thrown away. Figure 2 shows a distorted image and a fairly good undistorted image after lens distortion calibration. The linear and non-linear solutions in this case yielded very visually similar results.

![Fig. 2. Performance on real images: (a) input distorted image. (b) output undistorted image.](image)

5. CONCLUSIONS

In this paper, we have derived a novel distortion measure that can be used to calibrate the distortion parameters of the lens of a camera. We have proposed two methods to solve for the parameters that minimize this measure. The first method is a non-linear method, while the second one is a fast, linear method that can provide a closed-form solution to the distortion coefficients. This represents a major advantage of our approach over other existing techniques. We are currently conducting more experiments to evaluate this approach under different noise levels and on real data.

6. REFERENCES


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