A Complete Volumetric 3D Model of the Human Jaw


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Abstract. The purpose of this paper is to introduce a detailed approach for constructing a complete model of the human jaw. Our previous work developed an accurate model of the human jaw surface. Unfortunately this surface is not enough for the analysis purpose which requires a detailed model of the teeth and their roots. A database of volumetric 3D models of teeth will be constructed. The upper part of the tooth coming from the shape from shading algorithm provided in our previous work is matched with one of the database teeth. A non-rigid registration technique is used to deform the matched tooth with our previous derived surface model. Combining this information with the jaw details and x-ray information of the roots, we can now derive a complete volumetric 3D model of the human jaw. This model is suitable for the analysis process which will include Finite Element work to analyze the stress and strains of different simulations. The simulation processes will include tooth-implanting and alignment.

Keywords: Human jaw; registration; segmentation; level Set function.

1. Introduction

Orthodontic treatment involves the application of force systems to teeth over time to correct malocclusion. In order to evaluate tooth movement progress, the orthodontist monitors this movement by means of visual inspection, intra-oral measurements, fabrication of plastic models (casts), photographs and radiographs; a process, which is both costly and time consuming. Obtaining a cast of the jaw is a complex operation for the orthodontist, an unpleasant experience for the patient and may not provide all the details of the jaw. Dental radiography technology can provide the orthodontist with some 3-D information of the jaw. And while dental radiology is now widely accepted as a routine technique for dental examinations, the equipment is rather expensive and the resolution, though adequate for maxillofacial imaging, is still too low for detailed 3-D visualization. Furthermore, the dose required to enhance the resolution is unacceptably

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high. Some efforts have been devoted to computerized diagnosis in orthodontics, e.g., [1, 2]. Most of these 3-D systems for dental applications found in the literature rely on obtaining an intermediate solid model of the jaw (cast or teeth imprints) and then capturing the 3-D information from that model. User interaction is needed in such systems to determine the 3-D coordinates of fiducial reference points on a dental cast. Other systems that can measure the 3-D coordinates have been developed using either mechanical contact [2] or a travelling light principle [3] but result in only surface information. Likewise our previous work developed only a surface model of the entire jaw [4]. All of these techniques give the surface model of the entire jaw without any volumetric teeth and roots details.

The stress analysis of the human jaw requires a complete model of the teeth and roots details. These details can be achieved using CT scans which are expensive and the radiation dose is considered to be high and therefore not accepted as a routine practice.

The objective of this paper is to build a system that gives detailed volumetric information about the human jaw (see Fig. 1). This system will start from that surface model which is divided into individual crowns (the teeth upper parts). Each crown will be matched with a database of different individual teeth types. This match will give a complete model of the corresponding tooth that gives that maximum similarity measure. Local deformations will be applied to the selected database tooth using non-rigid registration to get a better match crown and root. X-ray images are used to deform roots. The objective of the database is to include the occluded root and teeth information in the 3D model of individual teeth. We can build this database from cone beamed CT scans of different subjects since the dose of this type of scans is acceptable. Those subjects will be classified based on age, race, and gender. Also micro CT scans can be used with manufactured models of the human jaw or cadavers. The surface information is combined to include to describe the complex topology and geometrical properties of the teeth surface.

![Fig. 1. A 3D finite element model of a manufactured jaw showing different mesh qualities (Very coarse, moderate and fine type meshes).](image-url)
Proposed Approach
We propose to assemble initially a limited database library of individual teeth models (see Fig. 2). Our complete 3D model (surface and root structure) model will then be enhanced and refined through algorithmic extraction of detailed features from the teeth library. The proposed approach consists of the following four main steps:

1. **Identification**: This will divide the given surface into individual teeth crowns.
2. **Matching**: For each tooth part, a suitable 3D complete model will be picked up from the library database using 3D surface rigid registration.
3. **Crown Matching**: In this step, the crown part of the selected model is registered to the tooth part in a free form deformation scheme.
4. **Root Matching**: In this step, the root part of the selected model is matched with an x-ray projection of the original teeth using non-rigid registration.

![Fig. 2. Some 3D models used in the database.](image)

Matching Step
The goal of image registration is to find the optimal transformation which aligns the given 3D data set with the database element model. To get accurate alignment we will use a combined transformation function $T$ which consists of a global transformation and a local transformation.

$$T = T_{\text{Global}} + T_{\text{Local}}$$  \hspace{1cm} (1)

The global transformation model describes the overall motion (translation and rotation) of the object. The simplest choice is a rigid transformation which is parameterized by 6 degrees of freedom, describing the rotations and translations of the object. A more general class of transformations are affine transformations, which have 6 additional degrees of freedom, describing scaling and shearing. In 3-D, an affine transformation can be written as:

$$T_{\text{Global}} = SR[x\ y\ z]^T + [t_x\ t_y\ t_z]^T$$  \hspace{1cm} (2)

The affine transformation captures only the global motion of the object (Scaling $S$, Rotation $R$, and Translation $t$). An additional transformation is required, which models the local deformation of the object. The nature of the local deformation of the object can vary significantly from one object to another object. Therefore, it is difficult to describe the local deformation via parameterized transformations. Instead, we have chosen an FFD model, based on B-splines [5], which is a powerful tool for modelling 3-D deformable objects and has been previously applied to the tracking and motion analysis in cardiac images [6]. The basic idea of FFD’s is to deform an object by manipulating an
underlying mesh of control points. The resulting deformation controls the shape of the 3-D object. To define a spline-based FFD, let \( \Psi \) denote an \( n_i \times n_i \) mesh of control points \( \Psi_{kl,k2} \) with uniform spacing. Then, the FFD can be written as the 3-D tensor product of the familiar 1-D cubic B-splines.

\[
T_{Local} = \sum_{l=0}^{3} \sum_{m=0}^{3} B_l(g_1)B_m(g_2)B_n(g_3)\Psi_{l+m+1,n+l+k}
\]

where \( i = [x/n_i - 1, j = [y/n_i - 1, k = [z/n_i - 1, g_1 = x/n_i - [x/n_i], g_2 = y/n_i - [y/n_i], g_3 = z/n_i - [z/n_i] \) and \( B_l \) represents the \( l^{th} \) basis function of the B-spline [5].

\[
B_0(g_1) = (1 - g_1^3)/6 \quad B_1(g_1) = (3g_1^3 - 6g_1^2 + 4)/6 \quad B_2(g_1) = (-3g_1^3 + 3g_1^2 + 3g_1 + 1)/6 \quad B_3(g_1) = g_1^3/6
\]

In general, the local deformation of the object should be characterized by a smooth transformation. To constrain the spline-based FFD transformation to be smooth, one can introduce a penalty term which regularizes the transformation. The general form of such a penalty term has been described by [7] as follows:

\[
\mathcal{E}_{\text{smooth}} = \frac{1}{V} \int_0^V \int_0^V \int_0^V \left( (\frac{\partial^2 T}{\partial x^2})^2 + (\frac{\partial^2 T}{\partial y^2})^2 + (\frac{\partial^2 T}{\partial z^2})^2 + (\frac{\partial^2 T}{\partial xy})^2 + (\frac{\partial^2 T}{\partial xz})^2 + (\frac{\partial^2 T}{\partial yz})^2 \right) dx dy dz.
\]
where $V$ denotes the volume of the image domain. To determine the correspondence between the given tag and the database element, we must define a similarity criterion which measures the degree of alignment between both volumes. A pixel-based similarity measure is mutual information (MI), which has been independently proposed in [8] and [9], and which has been shown to align images from different modalities accurately and robustly. Mutual information is based on the concept of information theory.

$$
\xi_{\text{Similarity}} (Y_1, Y_2) = H(Y_1) + H(Y_2) - H(Y_1, Y_2)
$$

(6)

where $H(Y_1), H(Y_2)$ are the entropies of the volumes $Y_1$ and $Y_2$. $H(Y_1, Y_2)$ denotes the joint entropy. If both images are aligned, the mutual information is maximized. It has been shown in [10] that mutual information itself is not independent of the overlap between two images. To find the optimal transformation, we minimize a cost function associated with the global transformation parameters, as well as the local transformation parameters. The cost function comprises two competing goals. The first term represents the cost associated with the image similarity in Eq. (6), while the second term corresponds to the cost associated with the smoothness of the transformation in Eq. (5).

$$
\xi = \xi_{\text{Similarity}} (Y_1, T(Y_2)) + \varepsilon \xi_{\text{Smooth}} (T)
$$

(7)

where $\varepsilon$ is a weighing coefficient which defines the trade off between the alignment of the two images and the smoothness of the transformation. We use the genetic algorithm in the optimization problem [11].

**Results**

Surface is divided into upper teeth parts used in the test. In Fig. 2, a part is registered with different database models. The feature taken for the mutual information is the surface point curvature. Three steps are shown with the initial positions, intermediate, and the final alignment step. It is clear that the fourth model is the best one that matches the given part based on the maximum mutual information. The last raw represents the final result of the non-rigid registration output. A shown in Fig.4, the x-ray scans can be used to get better match for the roots. The suggested system projects the model and measures the error with the x-ray image and based on this error, the model can change to give the best match.

![Fig. 4. Suggested validation system with x-ray scans.](image)
7. Conclusion and Future Work

We have proposed an approach of building a complete 3D model of the human jaw. This method uses a database of teeth for the purpose of matching. Our approach does not depend on a CT scan of each patient which is an expensive imaging. Registration techniques are used to deform the matched model to give the best fit with the available surface tooth upper part. X-ray imaging can be used to match the roots. The resultant model will allow accurate finite element analysis of the teeth in the human jaw. This analysis is very important especially in the teeth alignment and implanting process simulations.

References