Efficient Shape-based Segmentation using Level Sets

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Abstract

We propose a novel approach for shape-based segmentation of MR images based on a specially designed level set function format. This format permits us to better control the process of object registration which is an important part in the shape-based segmentation framework. The method depends on a set of training shapes used to build a parametric shape model. The intensity (gray level) is taken into consideration besides the shape prior information. The shape model is fitted to the image volume by registration through an energy minimization problem. The approach overcomes the conventional methods problems like point correspondences and weighing coefficients tuning of the partial differential equations. Also it is suitable for multi-dimensional data and computationally efficient. Results of extracting the 2D star fish and the brain ventricles, cerebellum, and corpus callosum in 3D demonstrate the efficiency of the approach.¹

Keywords: Level Sets, Image Segmentation, Partial Differential Equations (PDE’s), MRI Segmentation.

1 Introduction

Segmentation of anatomical structures is very important for medical visualization and diagnostics and still a challenging problem because of image noise, inhomogeneities, and lack of strong edges. Also many objects have similar gray level intensity values. Therefore this process cannot depend only on image information but also has to exploit the prior knowledge of shapes and other properties of structures to be segmented.

Level set segmentation techniques [1, 2, 3, 4] overcome problems of classical deformable models. A curve in 2D or a surface in 3D evolves in such a way as to cover a complex shape or structure. Its initialization is either manual or automatic but it need not be close to the desired solution. But these methods depend on a big number of parameters to be tuned for successful process. Different approaches (e.g. [5, 6]) were proposed to overcome the parameters tuning problem.

Different shape-based segmentation approaches are found in the literature. A 3D shape-based segmentation approach in [7] builds a shape model from a set of training shapes using distance functions. A level set function evolves minimizing the shape alignment energy and the intensity (gray level) one. But using a simple rigid transformation with a common scale s is insufficient especially when gathering training shapes from different patients’ scans. Also, the segmentation results are very sensitive to weighting coefficients of the partial differential equation (PDE).

In [8], a parametric shape model was introduced as weighted sum of signed distance maps. Parameters of a multi-shape model are calculated to minimize a single mutual information cost energy function. Binary images are used to align the training shapes; but this alignment process can easily get stuck with a local minima since the energy function depends on the differences of binary values.

Below we propose a novel more robust shape-based segmentation approach based on level set techniques depending on both intensity (gray level) and shape prior information. The approach includes training shapes registration and model building. Training curves/surfaces are collected to represent the variations of the target shape in the form of signed distance maps.

We introduce a new form of a signed distance map to handle complex rigid transformations with different scaling \((s_x, s_y, s_z)\), rotation \((\theta_x, \theta_y, \theta_z)\) and translation \((t_x, t_y, t_z)\) parameters of the shape registration. The previous work (e.g [7]) used conventional distance maps handling only the common scale

¹This study has been implemented at CVIP Lab., University of Louisville
(s = \mathbf{s}_x = \mathbf{s}_y = \mathbf{s}_z) with different rotations and translations. The distance maps result in more adequate energy function which is optimized to get the transformation parameters.

Also a shape-based PDE approach is introduced which includes no weighting coefficients to be tuned. In this approach, three different level set functions are used. The first function is built as a function of the signed distance maps of the training shapes in a form of a parametric shape model. The second one is the segmentation of the object of interest based on the intensity value. The last function represents the evolving shape resulting from the combination of the intensity and the shape information. An energy function is formulated to measure the difference between the shape model and the intensity functions. The shape and the pose parameters are required to minimize this energy in a gradient descent approach. Extracting brain ventricle, cerebellum, and corpus callosum from MR scans will be demonstrated to show the efficiency of the approach.

The paper is organized as follows. Section 2 presents the new level set formalism. Section 3 discusses the training shapes alignment and the evolution of the mean level set function. Representing the shape variations is discussed in Sec. 4. Section 5 shows the combined segmentation model. Experimental results are shown in Sec. 6, and conclusions and discussions are presented in Sec. 7.

This paper is an extension to the author’s work published at the ICCV-05 [9]: we provide evaluation and comparisons with other approaches. More practical examples and results in 2D and 3D are discussed. In addition, a closed form solution is used to calculated the weights that describe the shape model which makes the convergence more stable.

2 New Level Set Formalism and its PDE

Level sets are very convenient to represent shapes [10]. This representation is invariant to translation and rotation. We define a new level set function as a vector distance rather than a scalar conventional one. Given a 2D curve or a 3D surface \( V \) that represents boundaries of a certain shape, we can define the following level set function, \( \Phi : R^4 \rightarrow R^6 \) where \( \Phi(x, y, z, t) = [\phi(x, y, z, t), \phi_2(x, y, z, t), \phi_3(x, y, z, t)]^T \) and \( \phi_i(x, y, z, t) \) is defined as the projection of the minimum Euclidean distance between the point \( X = [x, y, z]^T \) and a curve/surface \( V \) in the \( i \)-th direction (we consider the 3D as the general case). The projections are negative inside the shape, positive outside, and zero on the boundary. If a rigid transformation with scales in different directions is applied to a given shape represented by the designed distance map, one can predict the distance map of the new shape by multiplying the transformed projection by the corresponding scale value. The registration details will come in the next section.

Within the level set formalism [11], the evolving curve/surface is a propagating front embedded as the zero level of a higher dimensional function \( \Phi \). It is straightforward to show that the continuous change of the projections of \( \Phi \) can be described by the following PDE:

\[
\frac{d}{dt} \phi_i + |\nabla \phi_i| F_i = 0, \quad i = 1, 2, 3. \tag{1}
\]

where \( F \) is a vector velocity function depending on the local geometric properties (local curvature) of the front and on the external parameters related to the input data e.g. image gradient. The hyper-curve \( \Phi \) deforms iteratively according to \( F \), and the position of the front is given at each iteration step by the equation \( |\Phi(x, y, z, t)| = 0 \). The design of the velocity function \( F \) plays the major role in the evolutionary process. Among several proposals in [12, 13], we have chosen \( F = [\nu - ek_1, \nu - ek_2, \nu - ek_3]^T \) where \( \nu = 1 \) or \(-1 \) for contracting or expanding the front, respectively, \( \epsilon \) is a smoothing coefficient always small with respect to 1, and \( k_i \) is the local curvature defined for the corresponding projection function \( \phi_i \) where \( i = 1, 2, 3 \). The latter parameter acts as a regularizing term.

3 Prior Knowledge and the Mean Shape

Let the training set consist of a set of training shapes \( V_1, ..., V_N \) with level set functions defined as above \( \Phi_1, ..., \Phi_N \). The goal is to calculate the set of pose parameters used to jointly align these shapes, and hence remove any variations in shape due to pose differences. The objective is to find a set of rigid transformations \( A_1, ..., A_N \) that register an evolving mean shape represented by \( \Phi_M \) to the given training shapes respectively. The method is different from that in [16] where the mean function is directly calculated as the average of the aligned distance maps. We assume that each transformation has scaling components \( s_x, s_y, s_z \), rotation angles \( \theta_x, \theta_y, \theta_z \) and translations \( T = [T_x, T_y, T_z]^T \). The scale matrix will be \( S = \text{diag}(s_x, s_y, s_z) \).

Registering the mean shape \( \Phi_M \) with the \( i \)-th shape, represented by \( \Phi_i \) means \( S_i\Phi_M \approx \Phi_i(A_i) \) where \( i = 1, N \) as shown above. So the transformations parameters should minimize the following energy function:

\[
E(\Phi_M, \Phi_1, ..., \Phi_N) = \sum_{i=1}^{N} \int_{\Omega} H(-\Phi_M) r_i^T r_i d\Omega \tag{2}
\]

where \( H(\cdot) \) is the Heaviside step function and \( r_i \) is the dissimilarity measure defined as follows:

\[
r_i = S_i\Phi_M - \Phi_i(A_i) \tag{3}
\]
Scaling will result in increasing/decreasing projections in each direction. The minimization of the energy with respect to the transformations parameters is done through the gradient descent as follows:

\[
\frac{d}{dt} s_i = 2 \int_\Omega H(-\Phi_M) r_i^T \nabla H(-\Phi_M)(A_i) \nabla \Phi_i d\Omega
\]

where \( s_i \in \{ s_x, s_y, s_z \} \) and \( a_i \in \{ T_x, T_y, T_z, \theta_x, \theta_y, \theta_z \} \) of transformation \( A_i \). We define \( r_{s_i} = \nabla \Phi_i (A_i) \nabla \Phi_M - \nabla \Phi_i (A_i) \nabla \Phi_M \).

The mean level set function \( \Phi_M \) evolves according its calculus of variations using the following PDE:

\[
\frac{d}{dt} \Phi_M = -\sum_{i=1}^{N} \int_\Omega \nabla \Phi_M^T \nabla H(-\Phi_M)r_i^T r_i d\Omega,
\]

where the change at the boundary is only considered by neglecting the term which contains \( H \) without derivative.

4 The Shape Model Level Set Function

The shape model is required to capture the variations in the training set. For this purpose, each curve/surface is transformed to the domain of the mean function \( \Phi_M \) by its corresponding transformation. The model is considered to be a weighted sum of the transformed signed distance maps deviated from the mean as follows:

\[
\Phi_p = \Phi_M + \sum_{i=1}^{N} w_i (\Phi_i - \Phi_M)
\]

where \( \Phi_i = S^{-1} \Phi_i (A_i) \) represents the transformed signed distance map marked by \( i \). We define \( w = [w_1 ... w_N]^T \) to be the weighting coefficient vector. By varying these weights, \( \Phi_p \) can cover all values of the training distance maps and hence the shape model changes according to all the given shapes. In [8], principal component analysis is used to get eigen shapes and a limited number of them is used to build the model. The shape variability is restricted to the selected eigen shapes while in the proposed approach all the training shapes are taken into consideration to enhance the results.

5 Segmentation Model

5.1 Adaptive Region Model

The intensity (gray level) segmentation is described by the function \( \Phi_g \) which changes according to Eq. (1). The term \( \nu_g = \pm 1 \) in the PDE specifies the direction of the front propagation. The problem can be reformulated as classification of each point at the evolving front (points of the narrow band region). If the point belongs to the associated object, the front expands, otherwise (the background) it contracts.

The point classification is based on the Bayesian decision [17] at point \( X \). The term \( \nu_g \) for each point is replaced by the function \( \nu_g (X) \) defined as follows:

\[
\nu_g (X) = \begin{cases} 
-1 & \text{if } \pi \cdot p_0 (I(X)) \geq \pi \cdot p_0 (I(X)) \\
1 & \text{otherwise}
\end{cases}
\]

where \( \pi \) is the region prior probability and \( p(.) \) is the corresponding probability density function (pdf) for the object \( o \) and the background \( b \). We characterize each region by a Gaussian distribution with adaptive parameters [6] as follows:

\[
\pi = \frac{\int_\Omega H(-\Phi_g) d\Omega}{\int_\Omega d\Omega}, \quad \pi_b = \frac{\int_\Omega H(-\Phi_b) d\Omega}{\int_\Omega d\Omega}
\]

\[
\mu_\pi = \frac{\int_\Omega H(-\Phi_g) I d\Omega}{\int_\Omega H(-\Phi_g) d\Omega}, \quad \mu_b = \frac{\int_\Omega H(-\Phi_b) I d\Omega}{\int_\Omega H(-\Phi_b) d\Omega}
\]

\[
\sigma^2_\pi = \frac{\int_\Omega H(-\Phi_g) (I - \mu_\pi)^2 d\Omega}{\int_\Omega H(-\Phi_g) d\Omega}, \quad \sigma^2_b = \frac{\int_\Omega H(-\Phi_b) (I - \mu_b)^2 d\Omega}{\int_\Omega H(-\Phi_b) d\Omega}
\]

5.2 Registering the Shape and Intensity Models

The shape model is fitted to the image volume by rigid registration using a rigid transformation \( A \) (defined as before) registering the intensity model \( \Phi_g \) to the shape model \( \Phi_p \). As stated in section 3, the registration is formulated as an energy minimization problem as follows:

\[
E(\Phi_g, \Phi_p) = \int_\Omega H(-\Phi_g) r^T r d\Omega.
\]

where \( r = S \Phi_g - \Phi_p (A) \) is the dissimilarity measure. The transformation \( A \) and the weight vector \( w \) should minimize the objective function \( E \); using the gradient descent approach as follows:

\[
\frac{d}{dt} \Phi_g = 2 \int_\Omega H(-\Phi_g) r^T \nabla H(-\Phi_g) (A) \nabla \Phi_g d\Omega
\]

\[
\frac{d}{dt} \Phi_p = \frac{d}{dt} \int_\Omega H(-\Phi_g) r^T (\Phi_p (A) - \Phi_M (A)) d\Omega,
\]

where \( s \in \{ s_x, s_y, s_z \} \) and \( a \in \{ T_x, T_y, T_z, \theta_x, \theta_y, \theta_z \} \) of the transformation \( A \).

5.3 A Closed Form for the Shape Parameters \( w_i \)

Calculating the shape parameters from the above equation is very sensitive to the initial values of the
weights. The initialization of these weights becomes so difficult specially when we have a big number of training shapes. The energy function is a quadratic function of the weights which leads to a closed form of the solution when the derivatives with respect to the weights are zeros as follows:

$$Cw = D$$ (14)

where $D$ is a column vector of size $N$ and $C$ is an $N \times N$ matrix. Their elements are calculated as follows:

$$D_i = \int_{\Omega} H(-\Phi_p)[S\Phi_p - \Phi_M(A)]^T r d\Omega, \quad (15)$$

$$C_{ij} = \int_{\Omega} H(-\Phi_p)[\Phi^t_j(A) - \Phi_M(A)]^T r d\Omega, \quad (16)$$

where $r = [\Phi^t_j(A) - \Phi_M(A)]$. Using individual training shapes (with variables not identical) guarantees that $C$ is a positive definite matrix avoiding singularity.

5.4 Combined Shape and Intensity Level Set Function

The segmentation involves both the intensity and shape prior information models. Another level set function $\Phi$ is used to represent the evolving region of the combined approach. It evolves according to Eq. (1) with the following directional term:

$$\nu = \left\{ \begin{array}{ll} -1 & \text{if } \Phi_p(A) \leq 0 \\ +1 & \text{if } \Phi_p(A) > 0 \end{array} \right. \quad (17)$$

If the pixel at the narrow band is mapped by the transformation $A$ inside the shape model based on the signed distance $\Phi_p$, the surface grows, otherwise it shrinks.

6 Experimental Results

6.1 Experiments in 2D

First we present some experiments in 2D to show the efficiency of the approach. Comparison of registering 2D shapes (corpus callosum and ventricles) is shown in figures 1 and 2. It is shown that using the new distance map representation give much better results than the conventional one.

We collected 200 images of the starfish to build its shape model. The shapes contours are extracted and then a signed distance map function is calculated for each one. Figure 3 shows the starfish, a sample of the training contours after registration, and the average shape model. The level set function that represent the average contour $\Phi_M$ is calculated through the training shapes registration. In Fig. 4-(a), a starfish image with horizontal strips is used in segmentation. The object edges are occluded with the strips and also with noise. The seeds of the intensity function $\Phi_p$ are initialized shown in red and the initial $\Phi$ is shown in yellow. Iteratively, the shape model signed distance map $\Phi_p$ is registered with $\Phi$ while the resulting segmentation contour $\Phi$ grows inside the shape model one. The segmentation result in Fig. 4-(c), shows that the shape model can successfully change and recover the corrupted object.

Figure 5-(a) shows another experiment. In this case, parts of the starfish are removed and horizontal strips are used in addition to noise. $\Phi_p$ and $\Phi$ are initialized in red and green respectively. The segmentation results in Fig. 5-(c) shows that the shape model can recover the removed parts and also the occluded edges.

6.2 MRI Brain Structures in 3D

The proposed segmentation approach is tested on the extraction of the ventricle, cerebellum, and corpus callosum from MR scans. We use 30 T1-weighted MRI data sets of size $256 \times 256 \times 125$, 20 data sets are used in the model building and the rest at the testing phase. The complexity of such models and the size variability in different directions motivate the use of more complex rigid transformations to register the training shapes than those in the previous works. Figure 6 shows how the proposed technique provides much better registration than the conventional one which cannot handle the size variability in different directions. More registration results in 3D are shown in Fig. 7.

Figure 8 exemplifies how the function $\Phi$ evolves for data set used in building the shape model. The results of the ventricles, cerebellum, and corpus callosum shows the ability of the model to change and extract object of interest. Figure 9 shows the difference between the radiologist’s segmentation and our results for a data set that is not used in building the shape model.

For the evaluation purpose, the brain ventricles of the above mentioned 31 data sets are segmented by a radiologist to be used as a ground truth. The error in Table 1 is calculated by measuring the actual difference between our segmentation and the ground truth. The boldfaced numbers represent the data sets that were not used in building the shape model. The mean and the standard deviation of error for the first 21 and the following 10 data sets are $(3.0\% \pm 1.51\%)$ and $(7.4\% \pm 3.4\%)$ respectively. The maximum error of $12.5\%$ occurs at the data set #24 which is not used in building the shape model.
Figure 1: Registration of two medical shapes in 2D and energies comparison. (a) The conventional results with the common scale $s$, and (b) Results using the proposed approach with the different scales $(s_x, s_y)$.

Table 1: The data sets numbers and the corresponding errors of extracting the brain ventricles.

<table>
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<th>01</th>
<th>02</th>
<th>03</th>
<th>04</th>
<th>05</th>
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<td>Error %</td>
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<td>3.0</td>
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<tr>
<td>Error %</td>
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7 Conclusions and Discussions

Our experiments confirmed the accuracy and robustness of the proposed approach that accounts for the intensity and shape prior information. The shapes are represented by the signed distance maps and a new level set function format is proposed to embed the complex rigid transformation to a new objective function. A new PDE system is used to deal with this latter function. The shape model is a function of the signed distance maps of the training shapes with different weights. The shape and the intensity models are registered iteratively to give the pose and weights parameters that minimize an objective function. Our approach overcomes the problems of the conventional ones. The proposed technique needs no point-wise correspondences between the training shapes and the PDEs have no weight parameters to tune. The approach is used to different image types. Extracting the 2D star fish and different anatomical structures of the brain with show the efficiency of the approach.

Acknowledgements

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References

Figure 2: Another example of two medical shapes in 2D and energies comparison. (a) The conventional approach results with the common scale $s$, and (b) Results using the proposed approach with the different scales $(s_x, s_y)$.

Figure 3: (a) The starfish image. (b), (c), (d), (e) A sample of the contours used in the training set. (f) The average shape contour.
Figure 4: (a) The initialization of the functions \( \Phi_1 \) (red) and \( \Phi_2 \) (yellow), (b) Intermediate stage of the evolution of the two functions, (c) The final segmentation result.

Figure 5: (a) The initialization of the functions \( \Phi_1 \) (red) and \( \Phi_2 \) (green), (b) Intermediate stage of the evolution of the two functions, (c) The final segmentation result.


Figure 6: Registration of two medical shapes in 3D and energies comparison. (a) Initial positions of two ventricles (dark and light gray), (b) Results using the conventional approach with the common scale $s$, and (c) The results using the proposed approach with the different scales $(s_x, s_y, s_z)$.


Figure 7: Registration results for 3D ventricles using the proposed approach, the initial positions (a) and (c), and the results in (b) and (d).

Figure 8: The evolution of $\Phi$ in different directions. (a) The ventricle evolving surface in 3D. (b), (c), (d) The projection of the evolving ventricle surface in different directions. (e), (f) Examples of the cerebellum and corpus callosum.
Figure 9: The evolution of $\Phi$ in different directions for extracting the brain ventricles of a data set that is not used in building the shape model. The segmentation of the radiologist is marked by red and ours by green.