GAUSSIAN CURVATURE FLOW MODEL FOR COLONIC POLYP DETECTION IN CT COLONOGRAPHY

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ABSTRACT

In this paper, we propose a novel anisotropic 3D surface evolution model for detecting protrusion shape based colonic polyp on the curved surface. The important feature of the proposed model is that it can detect protrusions with both convex and concave shapes. Protrusion shapes are defined as the extension beyond the usual limits or above a plane surface. Based on Gaussian and mean curvature flows, the approach works by locally deforming the convex or concave surface until the second principal curvature goes to zero. The diffusion directions are changed to prevent convex surfaces from converting into concave shapes, and vice versa. The deformation field quantitatively measures the amount of protrudeness. The proposed method has been evaluated by using synthetic phantoms and real colon datasets.

Index Terms— Colorectal Cancer, CT Colonography, Curvature Flow, 3D Shape Representation

1. INTRODUCTION

Colorectal cancer is the second leading cause of cancer-related death and the third most common form of cancer in the United States [1]. Since colonic polyps grow from the colon wall into the lumen air like dome structures, they are normally modeled as protrusion shapes in many literatures [2, 3, 4].

Different differential geometry based algorithms were proposed for protrusion shaped based automatic polyp detection. Yoshida and Nappi [2] used the curvature analysis to characterize polyps, folds and colon walls in the extracted colon. Finally, they applied fuzzy clustering to locate polyp. Summers et al. [3] investigated the feasibility of geometric features based shape analysis for polyp detection. Most of the current methods were dependent on thresholding different geometric features. Since they searched all the vertices (on the order of 1 million) on the triangle mesh surface of the real colon, it was really time consuming. Also, those methods modeled the polyps as symmetric shapes.

More recently, Wijk et al. [5] proposed an idea on surface evolution for protrusion detection, and applied to automatic polyp detection in CT colonography. Polyp candidate was considered as a local protrusion deviating from background. The points on convex parts of the protrusion iteratively moved inwards and finally the protrusion was flattened. Protrudeness was quantitatively measured by the displacement amount, and polyp candidates were detected by thresholding the displacement field. The method can locate the convex protrusion shapes with constraint \( \kappa_2 > 0 \), where \( \kappa_2 \) is the minimum principal curvature. However, only protrusions with convex shapes were detected by the only constraint, while those concave shapes were missed.

In this paper, we propose a novel anisotropic algorithm for 3D surface evolution to detect protrusion shape based colonic polyps. We have validated the proposed method using simulated phantom containing convex and concave synthetic polyps with different shapes and sizes. We also present the results using real colon datasets to demonstrate the effectiveness of the new surface evolution algorithm.

2. SURFACE REPRESENTATION BY CURVATURES

For a regular surface \( S \), Given the first fundamental forms \( E, F \) and \( G \), and the second fundamental forms \( L, M \) and \( N \) of surface \( S \), the Gaussian curvature \( K \) and mean curvature \( H \) can be defined as

\[
K = \kappa_1 \kappa_2 = \frac{LN - M^2}{EG - F^2} \tag{1}
\]

\[
H = \frac{1}{2} (\kappa_1 + \kappa_2) = \frac{EN - 2FM + GL}{2(EG - F^2)} \tag{2}
\]

where, \( \kappa_1 \) and \( \kappa_2 \) are the maximum and minimum principal curvatures.

Then the two principal curvatures can be computed as

\[
\kappa_1 = H + \sqrt{H^2 - K} \tag{3}
\]
Based on the two principal curvatures, 3D surface shape are represented by five different categories, and they are summarized in Table 1. Figure 1 shows some examples of 3D shape mesh surfaces.

3. SURFACE EVOLUTION FOR POLYP DETECTION

Desbrun et al. [6] proposed to use mean curvature flow to replace the Laplacian diffusion, thus the surface evolution becomes as:

$$\frac{\partial X_i}{\partial t} = -H_i \overrightarrow{n}_i$$  \hspace{1cm} (5)$$

where, $H_i$ and $\overrightarrow{n}_i$ are the mean curvature and the outer unit length normal vector at $X_i$, respectively.

In [6], the diffusion was applied to all mesh vertices. The surface moved along the normal vector direction at a speed proportional to the mean curvature, and finally achieved the desired smooth result with respect to the shape.

Wijk et al. [5] used Gaussian curvature flow for protrusion shape based colonic polyp detection. They applied the diffusion only to a limited number of mesh points with $\kappa_2 > 0$, instead of the entire mesh surface, to reduce the computational complexity greatly.

They introduced a ‘force’ term by minimizing the second principal curvature $\kappa_2$. The resulting equation becomes

$$L(X_i) = F_i(\kappa_2)$$  \hspace{1cm} (6)$$

The ‘force’ field initially balanced the displacement prescribed by the Laplacian and was updated by solving Eq. 6:

$$F_{i}^{t+1} = F_{i}^{t} - \kappa_2 \frac{A_{1-\text{ring}}}{2\pi} \overrightarrow{n}_i \text{ with } F_{i}^{t=0} = L(X_i)$$  \hspace{1cm} (7)$$

where, $A_{1-\text{ring}}$ is the surface area of the 1-ring neighborhood around mesh vertex $i$.

4. A NEW SURFACE EVOLUTION FORMULATION

The method mentioned above only worked for convex protrusion shape polyps, in this section, a new surface evolution formulation is presented to address this problem. First, besides the main constraint $\kappa_2 > 0$, we add two geometric features including shape index and curvvedness as strict detection descriptors to reduce computational complexity. Then, we propose an anisotropic formula for surface evolution to detect the general protrusions with elliptic convex, concave and even irregular shapes.

We consider constraints mainly on elliptic and hyperbolic points by simplifying the idea proposed in [7]. It aims to determine the appropriate moving direction of the velocity vector depending on two principal curvatures $\kappa_1$ and $\kappa_2$. The velocity diffusion directions based on Gaussian curvature flow are summarized in Table 2. Our goal of surface evolution for protrusion shape detection is that the algorithm deforms the local colon wall until the protrusions are flattened and the diffusion directions are changed to prevent convex surface from converting into concave shape, and vice verse. This idea is illustrated in Figure 2.

To achieve the above goal, we introduce the following dif-

\begin{table} [H]
\centering
\caption{Surface Shape Representation Using Curvature Analysis}
\begin{tabular}{ |c|c|c|c| }
\hline
$\kappa_2$ & $\kappa_1 < 0$ & $\kappa_1 = 0$ & $\kappa_1 > 0$ \\
\hline
$\kappa_2 < 0$ & elliptic concave ($H < 0$) & parabolic surface & hyperbolic surface ($H \neq 0$) \\
\hline
$\kappa_2 = 0$ & parabolic surface & plane ($H = 0$) & parabolic surface \\
\hline
$\kappa_2 > 0$ & hyperbolic surface ($H \neq 0$) & parabolic surface & elliptic convex ($H > 0$) \\
\hline
\end{tabular}
\end{table}
Table 2. Gaussian curvature flow based velocity diffusion direction

<table>
<thead>
<tr>
<th>Surface Shape Representation</th>
<th>Evolution Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptic Convex ($K &gt; 0, H &gt; 0$)</td>
<td>Moving Inward: $-\overrightarrow{n}$</td>
</tr>
<tr>
<td>Elliptic Concave ($K &gt; 0, H &lt; 0$)</td>
<td>Moving Outward: $\overrightarrow{n}$</td>
</tr>
<tr>
<td>Hyperbolic Surface ($K &lt; 0, H \neq 0$)</td>
<td>Moving Inward(or Outward): $-\overrightarrow{n}$ (or $\overrightarrow{n}$)</td>
</tr>
<tr>
<td>Parabolic ($K = 0$)</td>
<td>Not Moving</td>
</tr>
<tr>
<td>Others ($K \neq 0, H = 0$)</td>
<td>Not Moving</td>
</tr>
</tbody>
</table>

fusion equation:

$$\frac{\partial S}{\partial t} = \begin{cases} 
\text{sign}(H)K\overrightarrow{n} & \text{if } K > 0 \text{ and } H \neq 0 \\
\alpha K\overrightarrow{n} & \text{if } K < 0 \text{ and } H \neq 0 \\
0 & \text{if } K = 0 \text{ or } H = 0 
\end{cases} \quad (8)$$

As we know that under the discrete case, $H$ could be zero even if $H \neq 0$ under the continuous case. In order to avoid this instability, the Equation 8 takes the following format by adding strict constraint.

$$\frac{\partial S}{\partial t} = \begin{cases} 
\text{sign}(H)K\overrightarrow{n} & \text{if } K > 0 \text{ and } |H| \geq \epsilon \\
\alpha K\overrightarrow{n} & \text{if } K < 0 \text{ and } |H| \geq \epsilon \\
0 & \text{else}
\end{cases} \quad (9)$$

where, $|\alpha| < 1$ and $\epsilon > 0$.

Considering Equation 9, at every mesh vertex $i$, we propose a new anisotropic diffusion function for surface evolution in term of $\kappa_1$ and $\kappa_2$.

$$\frac{\partial X_i}{\partial t} = -\beta(\kappa_1, \kappa_2)\overrightarrow{n}_i \quad (10)$$

with,

$$\beta_i(\kappa_1, \kappa_2) = \begin{cases} 
e^{-1+\kappa_i}\text{sign}(H)K_i & \text{if } K_i > 0, |H_i| \geq \epsilon \\
e^{-1+\kappa_i}\alpha K_i & \text{if } K_i < 0, |H_i| \geq \epsilon \\
0 & \text{else}
\end{cases} \quad (11)$$

where, $1 \leq \gamma < \infty$.

After we introduce the new ‘force’ term based on $\kappa_1$ and $\kappa_2$, Equation 6 becomes

$$L(X_i) = \overrightarrow{F_i}(\kappa_1, \kappa_2) \quad (12)$$

By substituting the term $\kappa_2$ in Equation 7 by $\beta_i(\kappa_1, \kappa_2)$, we can get the resulting equation.

$$\overrightarrow{F_i}^t+1 = \overrightarrow{F_i}^t - \frac{A_1}{2\pi} \beta_i(\kappa_1, \kappa_2)\overrightarrow{n}_i \quad \text{with } \overrightarrow{F_i}=0 = L(X_i) \quad (13)$$

After mesh surface deformation, the displacement value is estimated by the following equation. Protrusion objects are located by thresholding the displacement field.

$$\text{disp}_i = |(P_{\text{final}})_i - (P_{\text{initial}})_i| \quad (14)$$

where, $(P_{\text{initial}})_i$ and $(P_{\text{final}})_i$ denote the positions of mesh vertex $i$ before and after mesh deformation, respectively.

5. VALIDATION AND RESULTS

In this section, we first validate the proposed 3D anisotropic surface evolution algorithm using synthetic cylindrical phantom with voxel size $1.0 \times 1.0 \times 1.0mm^3$. We assume that the surface normal is pointing into the cylinder phantoms and real colons. If the orientation of the entire isosurface is consistently defined, we can find three convex and one concave protrusions inserted at different locations as shown in Figure 3. The second convex protrusion shape is created as a ellipse-like protrusion with size $10 \times 12 \times 8mm^3$, while the other three spherical shapes are of sizes $20mm, 30mm$ and $10mm$, respectively. The proposed algorithm runs iteratively until all the protrusion shapes become nearly flat, which means all Gaussian curvatures of those the vertices go to zeros.

Figure 4 shows that the proposed 3D surface evolution algorithm works on convex protrusion shape. The results of the first convex protrusion shape shown in Figure 3 after 20, 50 and 100 iterations are shown in (b) through (d). The results of the concave protrusion shape are shown in (a) through (d) in Figure 5.

The final results for the whole phantom are illustrated in
6. CONCLUSION

In this paper, we have introduced a new anisotropic 3D evolution formula for general protrusion shapes detection. The algorithm incorporates Gaussian and mean curvature flows, and it deforms the local surface until the protrusions are flattened and the diffusion directions are changed to prevent convex surface from converting into concave shape, and vice versa. Both convex and concave protrusion shapes could be found by this formula, which is the main contribution in this paper.

7. REFERENCES


