A Novel Curvature Flow Based 3D Surface Evolution Formula for Polyp Detection

Dongqing Chen

Computer Vision & Image Processing (CVIP) Laboratory
Department of Electrical & Computer Engineering
University of Louisville
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Disadvantage of Geometric Feature Based Methods:
(1) Important assumption: polyp modeled as an approximately spherical or elliptical polypoid shape
(2) Search all the vertices on the meshed isosurface (~$10^6$).
(3) Depending heavily on different thresholds of curvatures, shape index, curvedness, etc.
3D Surface Shape Representation
Using Curvature Analysis

<table>
<thead>
<tr>
<th></th>
<th>$\kappa_1 &lt; 0$</th>
<th>$\kappa_1 = 0$</th>
<th>$\kappa_1 &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_2 &lt; 0$</td>
<td>elliptic concave ($H &lt; 0$)</td>
<td>parabolic surface</td>
<td>hyperbolic surface ($H \neq 0$)</td>
</tr>
<tr>
<td>$\kappa_2 = 0$</td>
<td>parabolic surface</td>
<td>plane ($H = 0$)</td>
<td>parabolic surface</td>
</tr>
<tr>
<td>$\kappa_2 &gt; 0$</td>
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<td>parabolic surface</td>
<td>elliptic convex ($H &gt; 0$)</td>
</tr>
</tbody>
</table>

$K = \kappa_1 \kappa_2$

$H = \frac{\kappa_1 + \kappa_2}{2}$

$K$: Gaussian Curvature

$H$: Mean curvature

$K_1$: the Maximum Principal Curvature

$K_2$: the Minimum Principal Curvature
Fig. 1. Examples of 3D surface shape by curvature analysis, (a) hyperbolic surface \((K < 0 \& H \neq 0)\), (b) elliptic convex surface \((K > 0 \& H > 0)\) and (c) elliptic concave surface \((K > 0 \& H < 0)\)
New Idea for Polyp Detection

1. Polyp bending on two directions, and folds bending only in one direction
2. For Polyp: $K_1 > K_2 > 0$, for folds: $K_1 > 0$, $K_2 \sim 0$
3. Reduce the computational complexity greatly, since only check the mesh vertices with $K_2 > 0$

C. van Wijk, V.F. van Ravesteijn, F.M. Vos, R. Truyen, A. de Vries, Detection of protrusions in curved folded surfaces applied to automated polyp detection in ct colonography. MICCAI 2006 471–478
New Idea for Polyp Detection

They only considered the convex protrusion shape based polyp with $K_2 > 0$. 
The Proposed New Algorithm

We consider the Gaussian curvature $K > 0$ and mean curvature, rather than $K_2$ only.

(a) convex shape

(b) concave shape

We consider the Gaussian curvature $K > 0$ and mean curvature, rather than $K_2$ only.
To achieve the above goal, we introduce the following diffusion equation:

\[
\frac{\partial s}{\partial t} = \begin{cases} 
\text{sign}(H)K \hat{n} & \text{if } K > 0 \text{ and } H \neq 0 \\
\alpha K \hat{n} & \text{if } K < 0 \text{ and } H \neq 0 \\
0 & \text{if } K = 0 \text{ or } H = 0
\end{cases}
\]
The Proposed New Algorithm

Under discrete case, $H$ could be zero even if $H \neq 0$ under continuous case. In order to avoid this instability, we add a strict constraint.

\[
\frac{\partial S}{\partial t} = \begin{cases} 
\text{sign}(H)K \vec{n} & \text{if } K > 0 \text{ and } |H| \geq \epsilon \\
\alpha K \vec{n} & \text{if } K < 0 \text{ and } |H| \geq \epsilon \\
0 & \text{if else}
\end{cases}
\]
The Proposed New Algorithm

Under discrete case or the triangulated mesh surface

\[ \frac{\partial X_i}{\partial t} = -\beta(\kappa_1, \kappa_2) \hat{n}_i \]  

(9)

with,

\[ \beta_i(\kappa_1, \kappa_2) = \begin{cases} 
  e^{-(1+\kappa_i^\gamma)} \text{sign}(H)K_i & \text{if } K_i > 0, \ |H_i| \geq \epsilon \\
  e^{-(1+\kappa_i^\gamma)} \alpha K_i & \text{if } K_i < 0, \ |H_i| \geq \epsilon \\
  0 & \text{else} 
\end{cases} \]  

(10)

where, \(1 \leq \gamma < \infty\).

New force function balancing the position of each vertex

\[ \overline{F}_{i}^{t+1} = \overline{F}_{i}^{t} - \frac{A_{1-ring}}{2\pi} \beta_i(\kappa_1, \kappa_2) \hat{n}_i \]  

with \( \overline{F}_{i}^{t=0} = L(X_i) \)
After mesh surface deformation, the displacement is estimated by the equation.

\[ disp_i = \left| (P_{final})_i - (P_{initial})_i \right| \]

where, \( (P_{initial})_i \) and \( (P_{final})_i \) denote the positions of mesh vertex \( i \) before and after mesh deformation, respectively.

And protrusion shape based colonic polyp candidates are located by thresholding the displacement field.
In this work, we assume that the surface normal is pointing into the cylinder phantom and real colons, and the orientation of the entire isosurface is consistently defined.
Some Preliminary Results
Some Preliminary Results
A new 3D surface evolution formula using Gaussian and mean curvature is proposed.

The proposed method works well on both convex and concave protrusion shape based colonic polyps.

More real dataset needed to test the robustness, true positive and false positive